VARIOUS IMPULSIVE SYNCHRONOUS PATTERNS FROM MUTUALLY COUPLED ISC CHAOTIC OSCILLATORS

Hidehiro Nakano†, Toshimichi Saito† and Kunihiro Mitsuhiro‡

†EEE Dept., HOSEI Univ., 3-7-2 Kajino-cho, Koganei-shi, Tokyo, 184 JAPAN
‡Maritime Safety Academy, Hiroshima, 737 JAPAN

ABSTRACT
Using two chaotic oscillators, a novel mutually coupled system is constructed, where the coupling is realized by impulsive signals. The system exhibits various synchronous phenomena. We have classified basic synchronous phenomena from the system and have clarified existence regions theoretically in the parameter space. These phenomena can be verified in the laboratory.

1. INTRODUCTION
Synchronization of chaotic oscillators have been studied extensively [1]-[6]. Such study is important as a basic nor linear problem. Also, it may open a way to new engineering applications, e.g., chaos-based communications [7] and neural networks [8]. In the study, the followings are basic important problems: (1) Classification of synchronous phenomena and (2) Recognition of synchronous phenomena. However, theoretical analysis of them is not sufficient so far. In order to approach to the problems theoretically, we introduce a simple coupling method for autonomous chaotic circuits, where the coupling is realized by impulsive signals. Applying the method to two impulsive switched capacitors (ISC) chaotic oscillators [9], we construct a mutually coupled system and consider its basic dynamics. The unit oscillator consists of linear elements and impulsive dependent switch. If a capacitor voltage hits a threshold, the capacitor voltage jumps to a base level instantaneously. In its mutually coupled system, each capacitor voltage jumps to each base levels instantaneously, if a capacitor voltage of either unit hits a threshold. Since the system is the piecewise linear model, the dynamics can be grasped using the piecewise exact solutions. The system exhibits various synchronous phenomena and we have classified basic synchronous phenomena:

- Master-slave synchronization of chaos and their coexistence phenomena
- Mutual synchronization of chaos
- Synchronization break down

Then we have clarified existence regions theoretically in the parameter space. These phenomena can be verified in the laboratory. In our previous works on the ISC systems [10][11], higher-order systems have been discussed mainly and the theoretical analysis has not been sufficient.

2. ISC CHAOTIC OSCILLATOR
Fig. 1 shows the ISC chaotic oscillator that will be an unit element in the coupled system. The dynamics is described by the following equation.

\[
\frac{d}{dt} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}, \quad \text{for } v_1 < V_T. \tag{1}
\]

If the capacitor voltage \( v_1 \) hits the threshold voltage \( V_T \), the dependent switch \( S \) is closed and the capacitor voltage \( v_1 \) jumps to the base voltage \( E(\leq V_T) \) instantaneously, where we assume that the conductance matrix in (1) has unstable complex eigenvalues. Using the following dimensionless variables and parameters, Equation (1) is transformed into the following normalized equation:

\[
\omega^2 = -\frac{g_{12}g_{22} - g_{11}g_{22}}{g_{11}g_{22}}, \quad \delta = \frac{g_{11} + g_{22}}{2g_{12}}, \quad p = \frac{g_{11} - g_{22}}{2g_{12}}.
\]

\[
\tau = \omega t, \quad b = \frac{\delta}{\omega}, \quad x = \frac{v_1}{V_T}, \quad y = \frac{V_{22} - v_2}{2g_{12}V_T},
\]

\[
\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \delta & 1 \\ -1 & \delta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}, \quad \text{for } x < 1. \tag{2}
\]

If \( x = 1 \), \((x,y) \rightarrow (b,y-p(1-b))\).
where \( \tau \) denotes differentiation by a normalized time \( \tau \). This system has three parameters: a damping \( \delta \), a jumping slope \( p \), and a jumping base \( b \). In Ref.[9], we have given a chaos generating condition in the parameter space, where we use definition of chaos in [12]: the embedded return map is ergodic and has a positive Lyapunov exponent. Hereafter we assume that the parameters \( (\delta, p, q) \) satisfy the condition. Fig.2(1) shows the typical chaotic attractor. The piecewise exact solution of (2) is given by

\[
\begin{align*}
(x, y) &= e^{\tau (\cos \tau \sin \tau - \sin \tau \cos \tau)} (x(0), y(0)),
\end{align*}
\]

for \( x < 1 \). (3)

In order to derive an 1-D return map, we define \( B = \{ (x, y) | x = b \} \) (see Fig.2(2)). Let a point on \( B \) be represented by its \( y \)-coordinate. The orbit started from a point \( y_n \) on \( B \) hits the threshold at the some positive time \( \tau_f \). It then jumps (and returns) to the base \( B \). Let \( y_{n+1} \) be the jump point. Then, we can define the following 1-D return map:

\[
\begin{align*}
F : B \rightarrow B, \quad y_n \rightarrow y_{n+1},
\end{align*}
\]

\[
\begin{align*}
y_{n+1} &= F(y_n) = e^{\tau_f (b \cos \tau_f + y_n \sin \tau_f)}.
\end{align*}
\]

In the mutually coupled system, the phase differential and amplitude rate between two units change only when both units jump to the base. The simultaneous jump is a key to classify synchrononous mechanism [11].

3. MUTUALLY COUPLED SYSTEM

Fig.3 shows the mutually coupled system. Using the following transformation of variables and parameters:

\[
\begin{align*}
b_M &= \frac{E_M}{V_T}, \quad b_S A = \frac{E_S A}{V_T}, \quad b_S B = \frac{E_S B}{V_T},
\end{align*}
\]

\[
\begin{align*}
x_A &= \frac{V_A}{V_T}, \quad x_B = \frac{V_B}{V_T},
\end{align*}
\]

the system dynamics is described by:

\[
\begin{align*}
\text{unit } A : \quad \begin{pmatrix} \dot{x}_A \\ \dot{y}_A \end{pmatrix} &= \begin{pmatrix} \delta & 1 \\ -1 & \delta \end{pmatrix} \begin{pmatrix} x_A \\ y_A \end{pmatrix}, \\
\text{unit } B : \quad \begin{pmatrix} \dot{x}_B \\ \dot{y}_B \end{pmatrix} &= \begin{pmatrix} \delta & 1 \\ -1 & \delta \end{pmatrix} \begin{pmatrix} x_B \\ y_B \end{pmatrix},
\end{align*}
\]

for \( x_A < 1 \) and \( x_B < 1 \). (5)

If \( x_A = 1 \), then \( (x_A, y_A) \rightarrow (-b_M, y_A - p(1+b_M)) \),

\( (x_B, y_B) \rightarrow (b_S B, y_B - p(x_B - b_S B)) \).

If \( x_B = 1 \), then \( (x_B, y_B) \rightarrow (-b_M, y_B - p(1+b_M)) \),

\( (x_A, y_A) \rightarrow (b_S A, y_A - p(x_A - b_S A)) \).

That is, each unit has two switches and two bases: the first switches \( (S_{A \delta} \text{ and } S_{B \delta}) \) are controlled by self capacitor voltages for self chaotic oscillation; and the second switches \( (S_{A \xi} \text{ and } S_{B \xi}) \) are controlled by the partner capacitor voltages for interaction. Fig.4 picks up typical phenomena calculated using the piecewise exact solutions:

a. In-phase synchronization of chaos:

either unit \( A \) or \( B \) is the master and the partner is the slave.

b. Anti-phase synchronization of chaos:

either unit \( A \) or \( B \) is the master and the partner is the slave.

c. Mutual synchronization of chaos:

both \( x_A \) and \( x_B \) can hit the threshold.

d. Synchronization breakdown.

Figure 3: Mutually coupled system

\( (x_A \propto V_{1A}, x_B \propto V_{1B}, b_M \propto E_M,\ b_{SA} \propto E_{SA}, b_{SB} \propto E_{SB}) \).
In this mutually coupled system, we give a basic classification of the phenomena:

<table>
<thead>
<tr>
<th>IMS-A</th>
<th>In-phase master-slave synchronization where unit A is the master.</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMS-B</td>
<td>In-phase master-slave synchronization where unit B is the master.</td>
</tr>
<tr>
<td>AMS-A</td>
<td>Anti-phase master-slave synchronization where unit A is the master.</td>
</tr>
<tr>
<td>AMS-B</td>
<td>Anti-phase master-slave synchronization where unit B is the master.</td>
</tr>
<tr>
<td>MU</td>
<td>Mutual synchronization.</td>
</tr>
<tr>
<td>BD</td>
<td>Breakdown.</td>
</tr>
</tbody>
</table>

We have obtained existence regions of these phenomena theoretically in the parameters space cross section of which is shown in Fig.5. Note that the above master-slave synchronizations are generated in the mutually coupled system. It may relate deeply to separation and/or clustering function from a viewpoint of artificial neural networks [8].

If this system shows the master-slave synchronization phenomenon, the slave attractor is similar to the master one and is smaller than that. The rate of the similarity is defined by:

\[
\alpha = \frac{\text{unit A}}{\text{unit B}} = \left\{ \begin{array}{ll}
\frac{b_M}{b_{SB}}, & \text{unit A = master} \\
\frac{b_{SA}}{b_M}, & \text{unit B = master}.
\end{array} \right.
\]

When \( \alpha > 0 \), this system shows in-phase master-slave synchronization (a). When \( \alpha < 0 \), it shows anti-phase master-slave synchronization (b). Since the slave cannot reach the threshold, the existence region of the master-slave synchronization is given by:

\[
-b_M < b_{SB} < b_M e^{-\pi}, \quad \text{unit A = master.} \tag{7}
\]

\[
-b_M < b_{SA} < b_M e^{-\pi}, \quad \text{unit B = master.} \tag{8}
\]

When both (7) and (8) are satisfied, synchronous states coexist. It means that the system exhibits either phenomenon depending on initial state. When neither (7) nor (8) is satisfied, each unit hits the threshold mutually, and the master-slave synchronization is broken down. Fig.6 shows hitting rate of unit A: it visualizes transition among master-slave synchronization state (rate=1 or 0), mutual synchronization state (rate=0.5) and the other states. For these results we have firstly checked stability of the synchronous state using piecewise exact solutions: calculating errors between the master and the slave states at each switching moment. Both units A and B jump on the base lines, when either unit hits the threshold. Let \( y_A, y_B \) be their y-coordinate, respectively. We then define the errors, \( \Delta y_h \), as the following:

\[
\Delta y_h = \left\{ \begin{array}{ll}
y_A + \frac{b_{SB}}{b_M} y_B, & x_A = b_M, \\
y_B + \frac{b_{SA}}{b_M} y_A, & x_B = b_M.
\end{array} \right.
\]

Time average of \( \Delta y_h \) is shown in Fig.7. As the stability can be checked, the master-slave similarity can be guaranteed. Note that mutual synchronization (c) can be confirmed for \( b_{SA} \approx b_{SB} \approx b_M \) in the figure. When such check is hard, we mark the parameter region as break down (d).
Figure 8: The implementation circuit based on the Wien bridge oscillator as unit element.

\[ v_i = V_i, \quad E \]

Figure 9: Laboratory experiments (1/g = 130kΩ, C = 2nF, k = 3.1, V_T = 1.0V, E_M = E_{SA} = -0.1V).

(a) in-phase master-slave synchronization (E_{sa} = -0.1V)

(b) anti-phase master-slave synchronization (E_{sa} = +0.1V)

(c) mutual synchronization (E_{sa} = +0.2V)

(d) break down (E_{sa} = +0.25V)

4. LABORATORY EXPERIMENTS

Using a Wien bridge oscillator with dependent switch, we have constructed an SC chaos oscillator as shown in Fig.8, where the threshold voltage is measured by a comparator. Preparing two pieces of the chaos oscillators, we can construct a mutually coupled system defined by Fig.3. Each unit is connected by only switching signals. Fig.9 shows laboratory experiments from this implemented circuit.

5. CONCLUSION

Mutually coupled system of the chaotic oscillators exhibits various synchronous phenomena. We classified and clarified the phenomena theoretically. These results are verified by laboratory experiments. In order to develop this study, we are considering (1) Classification of coupling strength, (2) Detailed classification of synchronous phenomena, (3) Bifurcation phenomena and (4) Extension to large scale systems.

6. REFERENCES


