A NETWORK OF RELAXATION OSCILLATORS BASED ON INTERMITTENTLY COUPLED CAPACITORS

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ABSTRACT

This paper presents \( N \) binary hysteresis relaxation oscillators connected by the Intermittently Coupled Capacitors. The network exhibits various interesting synchronous phenomena. As a powerful analysis tool, we derive a hybrid return map which has one real and \( N \) binary states. Using this map, the stability of the periodic synchronization can be analyzed simply by the binary states: it is not necessary to check the real state. Then we classify basic phenomena in a bifurcation diagram. Typical phenomena are verified in the laboratory.

1. INTRODUCTION

This paper presents binary hysteresis relaxation oscillators connected by the Intermittently Coupled Capacitors (ab. ICC) [1], and gives basic theoretical results. The ICC directly connects the capacitors (selected one by one from each oscillator). Then, at every connecting moment, all the capacitor voltages are equalized instantaneously. It exhibits various synchronous and asynchronous phenomena. In order to analyze the stability of periodic synchronization, we derive a Hybrid Return Map (ab. HRM) which has one real and \( N \) binary states, where \( N \) is the number of oscillators. The real state corresponds to the equalized states of the oscillators, and the binary states correspond to outputs of the hysteresis elements. The HRM is a powerful analysis tool: the stability of the periodic synchronization can be analyzed theoretically by the binary states of the HRM. That is, it is not necessary to check the real state. Since the HRM has the real state, it generates a larger variety of binary sequences than purely binary systems (e.g., shift register generators [2]). Then the ICC network may be developed into some engineering applications, e.g., dynamic associative memory and multi-bit ADC [3]. We apply the analysis tool to a basic example, two coupled oscillators, and demonstrate basic interesting periodic synchronous phenomena. They are classified into a bifurcation diagram. Typical phenomena are verified in the laboratory.

There exist various connection methods: connection by \( R \), \( L \), \( C \), \( M \), and so on. These networks exhibit various interesting synchronous phenomena. However, theoretical analysis is very difficult even for periodic synchronization [4]-[7]. In Ref.[1], we have considered the Occasional Linear Connection (ab. OLC) method for piecewise linear oscillators. The ICC can be regarded as a development of the OLC: the ICC can be applied not only for the piecewise linear oscillators but also for the smooth ones.

2. THE ICC NETWORK

Fig.1 shows a network of \( N \) hysteresis relaxation oscillators, where the connection is realized by the Intermittently Coupled Capacitors (ab. ICC). The \( i \)-th oscillator consists of one capacitor and one hysteresis Voltage Controlled Current Source (ab. VCCS):

\[
C_i \frac{dv_i}{dt} = H_i(v_i), \quad H_i(v_i) = \begin{cases} I_i, & \text{for } v_i \leq E, \\ -I_i, & \text{for } v_i \geq -E, \end{cases}
\]

where \( H_i(v_i) \) switches from \( I_i \) to \( -I_i \) (respectively, \( -I_i \) to \( I_i \)) if \( v_i \) hits \( E \) (respectively, \( -E \)). In this paper, we implement the hysteresis VCCS by the operational transconductance amplifier: \( I_i \) is the saturation current of them, and \( E = R_i I_i \). As shown in Fig.2(a), the trajectory changes its direction of motion when it hits the threshold. It exhibits periodic triangular waveform. In the ICC network, the oscillators are connected by the switches \( S_i, i = 1, 2, ..., N \), all of which are closed by the ICC signal \( S(t) \) at \( t^+ = nT^- \), \( n = 0, 1, ..., \) impulsively and simultaneously. At every switching moment, all the capacitor voltages are equalized instantaneously \( ^1 \):

\[
v_i(t) = \bar{v}(t) = \sum_{k=1}^{N} w_k v_k(t^-), \quad t^- = nT^-,
\]

where \( w_k = C_k / \sum_{k=1}^{N} C_k \). Fig.2(b) shows a measured waveform from the ICC network. Using the following dimensionless variables and parameters

\[
\tau = \frac{t}{T}, \quad x_i = \frac{v_i}{E}, \quad \lambda_i = \frac{EC_i}{T I_i}, \quad h(x_i) = \frac{1}{I_i} H_i(E x_i),
\]

the system dynamics is described by

\[
\begin{cases}
\lambda_i \frac{dx_i}{d\tau} = y_i, \quad y_i = h(x_i), \quad \text{for } \tau^- \neq n^-, \\
x_i(\tau) = \sum_{k=1}^{N} w_k x_k(\tau^-), \quad \text{for } \tau^- = n^-,
\end{cases}
\]

\( ^1 \) Such instantaneous switching is not correct in physical sense: energy continuity property is broken. However, this approximation based on Kirchhoff’s voltage law is accepted in the switched capacitor techniques.
be described explicitly. (a) The ICC network. (b) Implementation example of the ICC network of hysteresis relaxation oscillators. (b) Implementation example of the ICC network.

First we derive a hybrid return map from the unit oscillator as a preparation for the map from the ICC network. Letting $I_A \equiv \{(x, y, \tau) | \tau \equiv 0 \mod 1 \}$, the trajectories started from $I_A$ return into $I_A$. Hence we can define a return map $F_A$ from $I_A$ into itself:

$$F_A \equiv (f, g) : R \times B \rightarrow R \times B,$$

$$(x(n+1) = f(x(n), g(n)), \quad y(n+1) = g(x(n), y(n)), \quad where \ n = 0, 1, ... \ denotes \ the \ discrete \ time. \ f \ and \ g \ can \ be \ described \ explicitly.

If $l \leq T_p^{-1} < l + 0.5$,

$$f(x, 1) = \begin{cases} x + 4T_p^{-1} - 4l, & \text{for } -1 < x \leq x_b, \\ -x - 4T_p^{-1} + 4l + 2, & \text{for } x_b < x < 1, \end{cases}$$

$g(x, 1) = -\text{sgn}(x - x_b).

If $l + 0.5 \leq T_p^{-1} < l + 1$,

$$f(x, 1) = \begin{cases} -x - 4T_p^{-1} + 4l + 2, & \text{for } -1 < x \leq x_c, \\ x + 4T_p^{-1} - 4(l + 1), & \text{for } x_c < x < 1, \end{cases}$$

$g(x, 1) = \text{sgn}(x - x_c).

$F_A$ is a one-ratio piecewise function. If $\lambda \equiv \{(\bar{x}, \bar{y}, \bar{\tau}) | \bar{\tau} \equiv 0 \mod 1 \}$, the trajectories started from $I_C$ return into $I_C$. Then we can define a hybrid (one real and $N$ binary states) return map (HRM) $F_C$ from $I_C$. The parameter $\lambda$, controls each oscillation period.
be measured by their Hamming distance:

For simplicity, we consider the case of

Fig. 5(a) is stable and in Fig. 5(b) is not stable.

by the binary state vector

That is, the stability of an

The proof will appear in the fully developed version.

Theorem : Let the distance between \( \mathbf{y}(n) \) and \( \mathbf{y}(n - 1) \) be measured by their Hamming distance:

\[
D(\mathbf{y}(n), \mathbf{y}(n - 1)) = \# \{ k | y_k(n) \neq y_k(n - 1) \}.
\]

Also, let the HRM has an r-periodic point. The periodic point is stable if and only if there exists a positive integer \( n^* \) such that \( 0 < D(\mathbf{y}(n^*), \mathbf{y}(n^* - 1)) < N \), \( 0 < n^* \leq r \), in the periodic binary sequence vector \( \{ \mathbf{y}(0), ... , \mathbf{y}(r - 1) \} \).

The proof will appear in the fully developed version.

That is, the stability of an r-periodic point can be analyzed by the binary state vector \( \mathbf{y}(n) \) of the HRM without calculating \( d\overline{f}/d\overline{x} \). For example, a periodic orbit with \( \mathbf{y}(n) \) in Fig. 5(a) is stable and in Fig. 5(b) is not stable.

4. BASIC PHENOMENA

For simplicity, we consider the case of \( C_n = C \) hereafter:

\[
w_k = 1/N \quad \text{in Equation (4). Also, we consider two oscillators network: } \] \( N = 2 \). Then a synchronous phenomenon corresponding to 2-periodic point is stable if \( D(\mathbf{y}(1), \mathbf{y}(0)) = 1 \); and is not stable if \( D(\mathbf{y}(1), \mathbf{y}(0)) = 0 \) or 2, where \( \mathbf{y}(n) \in B^2 \). Here, as a basic phenomenon, we consider the stable synchronization corresponding to 2-periodic point as shown in Fig. 6(a): \( F_C^2(\overline{x}_p, \overline{y}_p) = (\overline{x}_p, \overline{y}_p) \). The corresponding transition of the binary state vector \( \mathbf{y}(n) \) of the HRM is shown in Fig. 6(b). This implies \( D(n) = 1 \). Then the network exhibits synchronization of stable periodic orbits. In order to characterize stable periodic orbits in the time-domain, we define a description method.

Definition 2: Let \( SW_{ICC} \) be the number of the ICC switchings during the time period of a stable periodic orbit corresponding to an r-periodic point, that is, \( SW_{ICC} = r \). Let \( SW_i \), \( i = 1, 2 \), be the number of switchings of the binary state \( y_i \) during the time period. Then we describe the synchronous phenomena of the periodic orbits by the ratio \( SW_{ICC} : SW_1 : SW_2 \). In Fig. 6(a), \( g_1(\tau) \) and \( g_2(\tau) \) switches twice and four times during one period, respectively, that is, \( SW_{ICC} = 2 ; \) \( SW_1 = 2 : 2 : 4 \). Fig. 6(c) shows a measured waveform with the ratio 2 : 2 : 8. Note that the HRM \( F_C \) from both Fig. 6(a) and Fig. 6(c) generate the same \( \mathbf{y}(n) \) in Fig. 6(b). Generally, the same \( \mathbf{y}(n) \) is generated by \( F_C \) when \( SW_{ICC} : SW_1 : SW_2 = 2 : 2 : 4m_1 : 4m_2 \), where \( m_1 \) and \( m_2 \) are positive integers. Fig. 7 shows plots of \( x \) for various \( T_{p1}^{-1} \). The dots (a) and (c) correspond to Fig. 6(a) and (c), re-
Figure 7: Plots of $\bar{x}(n)$ for various $T_{p2}^{-1}$. ( $T_{p1}^{-1} = 0.5$.) The dots indicated by (a) and (c) correspond to Fig.6(a) and (c), respectively.

Figure 8: Bifurcation diagram for various $SW_{ICC}$ which are indicated by the numbers. $l_1$ and $l_2$ are non-negative integers. The diagram for $0.5 < T_{p1}^{-1} - l_1 \leq 1$ (respectively, $0.5 < T_{p2}^{-1} - l_2 \leq 1$) is symmetric with respect to $T_{p1}^{-1} - l_1 = 0.5$ (respectively, $T_{p2}^{-1} - l_2 = 0.5$). (a), (c) and (d) correspond to Fig.6(a), Fig.6(c) and Fig.9, respectively. Even if the parameter $T_{p2}$ changes slightly from those dots, the ICC network exhibits a stable periodic orbit with the same switching ratio. Fig.8 shows a bifurcation diagram for various $SW_{ICC}$, where $l_1$ and $l_2$ are non-negative integers. The dots (a) ($l_1 = 0$, $l_2 = 1$) and (c) ($l_1 = 0$, $l_2 = 2$) correspond to the dots in Fig.7. It can be seen that this system exhibits various types of synchronous phenomena. An interesting phenomenon is shown in Fig.9 which corresponds to the dot (d) in Fig.8: the binary state vector $y(n)$ takes on all the possible combinations. Note that since the HRM has the real state $\bar{x}$, it has a potential to provide a larger variety of useful binary sequences than purely binary systems, e.g., the shift register generator.

5. CONCLUSIONS

We have studied a network of binary hysteresis relaxation oscillators, where the connection is realized by the ICC. As a powerful analysis tool, we have derived the HRM ($R \times B^N$ map). Then we can analyze the stability of periodic synchronization by the binary states of the HRM. That is, if it is not necessary to check the real state. Using the HRM, basic phenomena are classified into a bifurcation diagram. Typical phenomena are verified in the laboratory.

Future problems include (a) detailed analysis of bifurcation phenomena, (b) engineering applications (e.g., dynamic associative memory and ADC) and (c) design of the ICC network suitable for chip.

6. REFERENCES


