Basic dynamics from a pulse-coupled network of autonomous integrate-and-fire chaotic circuits

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Abstract

This paper studies basic dynamics from a novel pulse-coupled network (ab. PCN). The unit element of the PCN is an autonomous integrate-and-fire chaotic circuit having two states. For a basic master-slave PCN, we give basic classification of chaos synchronous phenomena and their breakdown patterns. Existence region of each phenomenon is elucidated in a parameter subspace. We then construct a large scale ring-type PCN, and elucidate that the PCN exhibits interesting grouping phenomena of chaos synchronization patterns. Some of typical phenomena can be verified in the laboratory.

1 Introduction

Integrate-and-fire models (ab. IFMs) are known as simple single neuron models, and have been studied intensively. Applying a sinusoidal stimulus, the IFM exhibits interesting periodic and aperiodic phenomena [1]-[4]. Using the IFMs, pulse-coupled network (ab. PCN) can be constructed [5]-[7]. The PCN exhibits a variety of synchronous and asynchronous phenomena. Analysis of the PCN is important not only as basic nonlinear problems but also for approach to information processing functions in the brain.

In this paper, we consider basic dynamics from a novel PCN in terms of nonlinear electronic circuits. First, we introduce an integrate-and-fire chaotic circuit (ab. IFC) that is a unit element of the PCN. The IFC consists of two capacitors, a linear 2-port voltage-controlled current source, and a firing switch: it is autonomous and has two state variables. Since the IFC is piecewise linear, we can elucidate that the IFC generates chaos in a theoretical sense [8]-[10]. Note that usual single state IFM can not generate chaos without the stimulus. Second, we construct a master-slave type PCN of 2 IFCs, and give a basic classification of the chaos synchronous phenomena and their breakdown patterns. The stability of the synchronous phenomena can be confirmed using the piecewise exact solutions. Existence region of each phenomenon is elucidated in a parameter subspace. Preliminary results can be found in [11] [12]. Note that complicated synchronous phenomena are hot topics in the study of coupled oscillators [13] [14], however, the analysis is hard because of complex nonlinearity. Third, we develop the master-slave PCN into a ring-type PCN and elucidate that the PCN exhibits interesting grouping phenomena of various chaos synchronization patterns. Plural grouping patterns co-exist and the PCN exhibits one of them depending on the initial states.

We have implemented a test circuit and some of typical phenomena can be verified in the laboratory. Note that our circuit design is well suited for realization on chips: it might be developed into a PCN-chip with efficient classification functions.

2 Integrate-and-fire chaotic circuit

In this section, we introduce the IFC as shown in Fig.1(a). It will be a unit element in the PCN. In the figure, the two-port voltage-controlled current source and two capacitors construct a linear oscillator. Then the operation of the firing switch $S$ is "As the capacitor voltage $v_1$ reaches the threshold voltage $V_T$, $S$ is closed and $v_1$ is reset to the base voltage $V_B$ instantaneously, holding $v_2=\text{const}$."

The circuit dynamics is described by Equation (1).

$$C \frac{d}{dt} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}, \text{ for } v_1 < V_T, \quad (v_1(t^+), v_2(t^+)) = (V_B, v_2(t)), \text{ if } v_1(t) = V_T, \quad (1)$$
and $x < 1$, the piecewise exact solution of Equation (3) is given by

$$p_{\text{jumping slope}}$$

The trajectory moves as illustrated in Fig.1(c). This system has three parameters: the damping $\delta$, the jumping slope $p$, and the jumping base $q$. For simplicity, we symbolize the IFC as shown Fig.1(b). For $x < 1$, the piecewise exact solution of Equation (3) is given by

$$\begin{bmatrix} x \\ y \end{bmatrix} = e^{\delta \tau} \begin{bmatrix} \cos \tau & \sin \tau \\ -\sin \tau & \cos \tau \end{bmatrix} \begin{bmatrix} x(0) \\ y(0) \end{bmatrix}$$

Equation (1) is transformed into Equation (3).

$$\left( \begin{array}{c} \dot{x} \\ \dot{y} \end{array} \right) = \left( \begin{array}{cc} \delta & 1 \\ -1 & \delta \end{array} \right) \left( \begin{array}{c} x \\ y \end{array} \right), \quad \text{for } x < 1, \quad \left( x(\tau^+), y(\tau^+) \right) = \left( q, y(\tau) - p(x(\tau) - q) \right), \quad \text{if } x(\tau) = 1,$$

where $(x(0), y(0))$ denotes an initial state vector at $\tau = 0$. As shown in Fig.1(c), the trajectory started from $L = \{(x, y) | x = q\}$ rotates divergently around the origin and reaches the threshold at the some positive time.

At the moment, the trajectory jumps onto $L$. Repeating this manner, the IFC exhibits chaotic attractor. Using a test circuit, we can confirm chaotic attractor in the laboratory as shown in Fig.1(d). The test circuit is implemented by an Op.Amp (TL074), a comparator (LM339), a mono-stable multivibrator (4538) and an analog switch (4066). In Ref.[12], we have derived 1D return map and have given theoretical evidence for chaos generation. Basic bifurcation phenomena are investigated. Also, a basic chip design of the IFC can be found in [9].

### 3 Master-slave PCN

Using two IFCs, we construct a master-slave PCN as symbolized in Fig.2(a). The dynamics is described by

$$\begin{bmatrix} x_A \\ y_A \end{bmatrix} = \left( \begin{array}{cc} \delta & 1 \\ -1 & \delta \end{array} \right) \begin{bmatrix} x_A \\ y_A \end{bmatrix}, \quad \begin{bmatrix} x_B \\ y_B \end{bmatrix} = \left( \begin{array}{cc} \delta & 1 \\ -1 & \delta \end{array} \right) \begin{bmatrix} x_B \\ y_B \end{bmatrix}, \quad \text{for } x_A < 1 \text{ and } x_B < 1,$$

$$\begin{bmatrix} x_A(\tau^+) \\ y_A(\tau^+) \end{bmatrix} = \left( q, y_A(\tau) - p(x_A(\tau) - q) \right), \quad \begin{bmatrix} x_B(\tau^+) \\ y_B(\tau^+) \end{bmatrix} = \left( b, y_B(\tau) - p(x_B(\tau) - b) \right), \quad \text{if } x_A(\tau) = 1,$$

$$\begin{bmatrix} x_B(\tau^+) \\ y_B(\tau^+) \end{bmatrix} = \left( q, y_B(\tau) - p(x_B(\tau) - q) \right), \quad \text{if } x_B(\tau) = 1.$$
Figure 2: Master-slave PCN, (a) Symbolized model, (b) Existence region of the synchronous phenomena, (c) Typical synchronous phenomena ($\delta \approx 0.05, p \approx 1.0, q \approx -0.2$): (1) $\alpha \approx 0.5$ ($b \approx -0.1$), (2) $\alpha \approx 0$ ($b \approx 0$), (3) $\alpha \approx -0.5$ ($b \approx 0.1$)

slave. Note that Unit B has double bases: the primary base $q$ is used for the self firing and the secondary base $b$ is used for the slave firing caused by the master. In the circuit, the master is the IFC in Fig.1(a) and the slave can be realized by adding a set of a switch and a base voltage source to the left capacitor of the IFC in parallel. The additional switch is closed as the master fires and the left capacitor voltage is reset to the base voltage corresponding to the parameter $b$. For simplicity, we fix $(\delta, p) = (0.05, 1.0)$ and select $(q, b)$ as control parameters. Although this impulsive coupling method is very simple, this system can exhibit a variety of synchronous and asynchronous phenomena. We can check the stability of the synchronous state theoretically using the average error expansion ratio between the master and the slave. If $q_t < q < 0$ ($q_t \approx -0.62$), the slave phase approaches to the master one. And if the slave attractor can not reach the threshold, we can observe the master-slave synchronization of chaos. As suggested in Fig.2(c), the slave attractor is similar to the master one. The similarity rate is given by $\alpha = x_B/x_A = y_B/y_A = b/q$. As $|\alpha|$ increases sufficiently, the state of the slave can reach the threshold and the synchronization is broken down. Based on the rate $\alpha$, we can classify the synchronous and asynchronous phenomena for $q_t < q < 0$:

1. $0 < b/q < 1$: In-phase master-slave chaos synchronization
2. $b/q = 0$: The master inhibits the oscillation of the slave.
3. $1/x_{A_{min}} < b/q < 0$: Anti-phase master-slave chaos synchronization
4. $b/q \geq 1$: In-phase like synchronization breakdown
5. $b/q \leq 1/x_{A_{min}}$: Anti-phase like synchronization breakdown

where $x_{A_{min}}$ is the minimum value of $x_A$ in the master attractor. We can clarify the basic mechanism of the breakdown theoretically: it will be discussed in the fully developed version. Existence regions of (1) to (5) can be summarized in the parameter subspace as shown in Fig.2(b).
4 Ring-type PCN

In this section, we consider an approach to large scale PCNs. As a first step, we consider a ring-type PCN consisting of 6 IFCs as illustrated in Fig.3(a). The dynamics is described by

\[
\begin{pmatrix}
\dot{x}_i \\
\dot{y}_i 
\end{pmatrix} = \begin{pmatrix}
\delta & 1 \\
-1 & \delta 
\end{pmatrix} \begin{pmatrix}
x_i \\
y_i 
\end{pmatrix}, \quad \text{for } x_i < 1, x_j < 1 \text{ and } x_k < 1,
\]

\[
(x_i(\tau^+), y_i(\tau^+)) = (q, y_i(\tau) - p(x_i(\tau) - q)),
\]

\[
(x_j(\tau^+), y_j(\tau^+)) = (b, y_j(\tau) - p(x_j(\tau) - b)),
\]

\[
(x_k(\tau^+), y_k(\tau^+)) = (b, y_k(\tau) - p(x_k(\tau) - b)), \quad \text{if } x_i(\tau) = 1,
\]

\[i = 1 \sim 6, \quad j = ((i + 1) \text{ mod } 6) + 1, \quad k = ((i - 1) \text{ mod } 6) + 1.\]

Note that each IFC is connected to its closest neighbor in bi-direction: this PCN is mutually coupled oscillators. As the master-slave PCN, we select \((q,b)\) as control parameters and set them in the shaded region in and Fig.2(b). In this case, referring to the result in Section 3, we notice that two groups of master-slave synchronization emerge as shown in Fig.3(b)-(d). Two masters are located in the diagonal position in the PCN, and their closest neighbor IFCs become the slaves. Then the 6 IFCs are separated into two groups and each group exhibits in-phase or anti-phase chaos synchronization depending on the parameters. Each of three diagonal pairs of the IFCs can be the master and three groupings co-exist as shown Fig.3(b)-(d). The PCN exhibits one of them depending on the initial state. This result can be extended easily to ring-type PCNs consisting of 3N IFCs. This grouping phenomenon seems to be a base for classification functions of the PCNs. It might be developed into spatial pattern generation or self-organizing functions in more generalized PCNs.

5 Conclusion

We have studied pulse-coupled integrate-and-fire type chaotic circuits. We have given a basic classification of the chaos synchronous phenomena and their breakdown patterns using a basic master-slave type PCN. The results have been summarized into a diagram in the parameter subspace. We have constructed a ring-type PCN that can exhibit interesting grouping phenomena of various chaos synchronization patterns.

Now we are considering the following problems: (1) Classification of the coupling methods, (2) More detailed classification of the phenomena, (3) Analysis of bifurcation phenomena, (4) Extension to more generalized systems, and (5) Application to flexible artificial neural networks.
References


