Synchronization and Chaos in Multiple-Input Parallel DC-DC Converters with WTA Switching

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SUMMARY This paper studies nonlinear dynamics of a simplified model of multiple-input parallel buck converters. The dynamic winner-take-all switching is used to achieve N-phase Synchronization automatically, however, as parameters vary, the synchronization bifurcates to a variety of periodic/chaotic phenomena. In order to analyze system dynamics we adopt a simple piecewise constant modeling, extract essential parameters in a dimensionless circuit equation and derive a hybrid return map. We then investigate typical bifurcation phenomena relating to N-phase synchronization, hyperchaos, complicated superstable behavior and so on. Ripple characteristics are also investigated. 

key words: synchronization, chaos, bifurcation, parallel dc-dc converters

1. Introduction

Parallel dc-dc converters (PDCs) are interesting study objects from both practical and fundamental viewpoints. The PDCs have common advantages of parallel systems such as improvement of reliability and fault tolerance. Roughly speaking, two classes of PDCs have been studied recently: single-input PDCs and multiple-input PDCs. The single-input PDCs have been considered mainly for lower voltages with higher current capabilities in the next generation microprocessors [1]–[8]. In order to reduce size and losses of the filtering stages, sharing output current with the lower ripple is required. The multiple-input PDCs have been considered mainly for clean energy power supplies such as solar-cells, wind generators, fuel cells and hybrid systems of them [9]–[11]. In such systems, stable power supplies from sources with different power capacities is required. In single- and multiple-input PDCs, several switching control techniques have been considered for efficient power supplies: digital logical control [1], sliding surface control [3], wireless PWM control [4], dynamic WTA-switching [8] and so on. On the other hand the PDCs are nonlinear dynamical system having rich phenomena [12]–[14]. For example, PDCs exhibit multi-phase synchronization that is basic to achieve current sharing with the lower ripple. As parameters vary, the synchronous phenomena can be changed into a variety of periodic/chaotic phenomena. However, analysis of such phenomena is not sufficient as compared with single dc-dc converters [15]–[22].

This paper studies nonlinear dynamics of a simplified model of multiple-input PDCs. In the PDC, N dc sources are applied to one load via N buck converters and the parallel coupling is realized through dynamic winner-take-all (WTA) switching. The WTA-switching can achieve N-phase Synchronization (N-SYN) automatically. First, we simplify the circuit dynamics into a piecewise constant (PWC) model and derive a dimensionless circuit equation. The PWC equation can describe both multiple- and single-input PDCs and has piecewise linear trajectories: it is well suited for unified and precise analysis [8], [15], [22]. Second, we derive hybrid return map (HRM) of N continuous variables and N binary variables. The HRM is useful to visualize/investigate various system behavior in both continuous conduction mode (CCM) and discontinuous conduction mode (DCM). Using the HRM we have investigated typical phenomena: bifurcation from N-SYN to hyperchaos [23] in CCM, distortion of N-SYN and bifurcation to chaos in CCM and bifurcation to complicated superstable phenomena due to the imbalance of the inputs. Ripple characteristics are also investigated. These results provide basic information to consider practical circuits design and to develop novel bifurcation theory.

Note that our previous paper [8] dose not discuss HRMs, multiple-input PDCs and bifurcation phenomena. Preliminary results of this paper can be found in [24].

2. Parallel Buck Converters

Figure 1 shows the multi-input PDC consisting of N buck converters (N ≥ 2) and the output current is shared \( i_o = \sum_{j=1}^{N} i_j \). If all the inputs have the same value, \( V_1 = \cdots = V_N \) then this system is equivalent to the single-input PDC studied in [8]. The \( j \)-th converter has a switch \( S_j \) and an ideal diode \( D_j \) which can be either of the three states:

State 1: \( S_j \) conducting, \( D_j \) blocking and \( 0 < i_j < J \)
State 2: \( S_j \) blocking, \( D_j \) conducting and \( 0 < i_j < J \)
State 3: \( S_j \) and \( D_j \) both blocking and \( i_j = 0 \)

Switching among them is defined by the rule

State 1 \( \rightarrow \) State 2 if \( i_j = J \)
State 2 \( \rightarrow \) State 3 if \( i_j = 0 \)
State 2 or State 3 \( \rightarrow \) State 1 if \( i_j = \min \) at \( t = nT \)

where \( J \) is a threshold current and \( T \) is a clock period. The dynamic WTA is used in switching to State 1: if \( i_j \) is the minimum among \( i_1 \) to \( i_N \) at \( t = nT \) then \( S_j \) is closed for
nT \leq t < (n + 1)T \text{ regardless of past situation of } S_j. \text{ We refer to the minimum } i_j \text{ as the winner at } t = nT. \text{ Plural winners are possible only on State 3 where } i_j(nT) = 0 \text{ must be minimum. If some converter operates to (not to) include State 3, it is said to operate in DCM (CCM). Note that the } N \text{ converters are connected through the WTA-switching: if the WTA-switching is not present, this system is to be } N \text{ independent single converters.}

For simplicity we assume } RC \gg T \text{ and replace the load with a constant voltage source } V_o < V_j. \text{ We also assume that all the circuit elements are ideal and } L_j = L. \text{ The circuit dynamics is described by Eq. (1) and the switching rule:}

\[
\frac{dL}{dt} = \begin{cases} V_j - V_o & \text{for State 1} \\ -V_o & \text{for State 2} \\ 0 & \text{for State 3} \end{cases}
\]

(1)

This is the PWC system having piecewise linear trajectory \cite{8,15,22}. Using the dimensionless variables and parameters:

\[
\tau = \frac{T}{L}, x_j = \frac{i_j}{I^*}, a_j = \frac{T}{L}(V_j - V_o), b = \frac{T}{L} V_o,
\]

(2)

the circuit dynamics is normalized into

\[
\frac{dx_j}{d\tau} = \begin{cases} a_j & \text{for State 1} \\ -b & \text{for State 2} \\ 0 & \text{for State 3} \end{cases}
\]

(3)

State 1 \rightarrow \text{State 2 if } x_j = 1

State 2 \rightarrow \text{State 3 if } x_j = 0

State 2 or State 3 \rightarrow \text{State 1 if } x_j \text{ wins at } \tau = n

where “x_j wins” means that x_j is the minimum in \{x_1, \ldots, x_N\}. Note that 0 \leq x_j \leq 1 is satisfied for \tau \geq 0. This system has } N + 1 \text{ positive parameters } a_1, \ldots, a_N, b \text{ and includes the PWC model of single-input PDCs \cite{8} in the special case } a_j = a. \text{ Here we recall three basic definitions in \cite{8}.}

\[\text{Definition 1:} \text{ Let } \mathbf{x} = (x_1, \ldots, x_N). \text{ The PDC is said to exhibit N-SYN if Eq. (3) has periodic solution with period } N \text{ such that } x(\tau + N) = x(\tau) \text{ and each converter becomes a winner once during one period } 0 \leq \tau < N.\]

\[\text{Definition 2:} \text{ Let } x_p = (x_{p1}, \ldots, x_{pN}) \text{ be a solution of N-SYN. The N-SYN is said to be stable for the initial state if } x(\tau) \text{ converges on } x_p(\tau) \text{ as time goes for } x(0) = x_p(0) + \epsilon(0) \text{ where } \epsilon(0) \text{ is a small initial perturbation. The N-SYN is said to be superstable for the initial state if } x(\tau) \rightarrow x_p(\tau) \text{ for } \tau > \tau_f \text{ and } x(0) = x_p(0) + \epsilon(0) \text{ where } \tau_f \text{ is some finite positive time.}\]

\[\text{Definition 3:} \text{ For a periodic solution with period } M, x(\tau + M) = x(\tau), \text{ ripple factor is given by } R_p = (\max X(\tau) - \min X(\tau))/\bar{X}(\tau), \text{ where } 0 \leq \tau < M, \bar{X}(\tau) \equiv \sum_{j=1}^{N} x_j(\tau) \text{ is the dimensionless output current and } \bar{X}(\tau) \text{ is its time average.}\]

The WTA-switching can realize N-SYN automatically and Fig. 2 shows some examples for \(N = 3\) where x_1, x_2 and x_3 can be the winner once during one period \(0 \leq \tau < 3.\) As suggested in Fig. 2(b), zero-ripple \(R_p = 0\) can be achieved in the special case \(a_1 = a_2 = a_3 \equiv a \) and \(2a = b \) or \(a = 2b.\) As \(a_3\) and/or \(b\) vary the 3-SYN is distorted as suggested in Figs. 2(c) and (d). In Sect. 3, it is shown that 3-SYN in Fig. 2(a) is stable but 3-SYN in Fig. 2(b) is unstable for the initial state. We can not observe unstable 3-SYN. It is also

![Fig. 1 Multiple-input parallel dc-dc buck converters.](image_url)

![Fig. 2 Typical shapes of 3-SYN (N = 3). (a) Stable 3-SYN in CCM for \(a_1 = a_2 = a_3 = 1.5\) and \(b = 3.3\) (\(R_p = 0.03\)). (b) Unstable 3-SYN in CCM for \(a_1 = a_2 = a_3 = 6.6\) and \(b = 3.3\) (\(R_p = 0\)). (c) Distorted 3-SYN in CCM for \(a_1 = a_2 = a_3 = 1.5\), \(a_3 = 2.6\) and \(b = 3.3\) (\(R_p = 0.13\)). (d) Distorted 3-SYN in DCM for \(a_1 = a_2 = 1.2\), \(a_3 = 0.4\), \(b = 2.4\) (\(R_p = 0.46\)). Parameter values are given for \(a_1, a_2, a_3, b\).](image_url)
shown that 3-SYN in DCM is superstable. Roughly speaking, superstable periodic orbit having the strongest attraction: it correspond to a fixed point with zero-slope in a 1D map [22].

3. Hybrid Return Map

In order to investigate system behavior we derive the HRM of \( N \) continuous and \( N \) binary variables. The HRM is the sampled state model as observed at every clock instant \( \tau = n \), where the state \( x_j(\tau) \) and \( \text{sgn}(\dot{x}_j(\tau)) \equiv s_j(\tau) \) at the \( (n+1) \)-th clock instant are expressed as a function of that at the \( n \)-th instant. Here we classify trajectory of \( x_j \) between two consecutive clock instants, \( n \leq \tau < n + 1 \), into five types as shown in Fig. 3.

Type 1: \( x_j \) increases without reaching \( x = 1 \) (CCM). \( a_j^{-1} > 1 \), \( 0 \leq x_j(n) < a_j \) and \( s_j(n) = 1 \) are required where \( a_j \equiv 1 - a_j \). Performing elementary geometrical calculations we obtain

\[
x_j(n+1) = x_j(n) + a_j, \quad s_j(n+1) = 1
\]  

(4)

Type 2: \( x_j \) increases, reaches 1 and decreases without reaching \( x = 0 \) (CCM). \( a_j^{-1} + b_j^{-1} > 1 \), \( a_j \leq x_j(n) < b_j \) and \( s_j(n) = 1 \) are required where \( b_j \equiv 1 - a_j + a_j/b_j \).

\[
x_j(n+1) = -p_j x_j(n) + q_j,
\]

\[
s_j(n+1) = \begin{cases} 
1 & \text{if } x_j(n+1) \text{ wins} \\
-1 & \text{otherwise}
\end{cases}
\]  

(5)

where \( p_j \equiv b/j \) and \( q_j \equiv 1 - b + p_j \).

Type 3: \( x_j \) increases, reaches 1, decreases and reaches \( x = 0 \) (DCM). \( b_j^{-1} < 1 \), \( b_j \leq x_j(n) \) and \( s_j(n) = 1 \) are required.

\[
x_j(n+1) = 0, \quad s_j(n+1) = 1
\]  

(6)

Type 4: \( x_j \) decreases and reaches \( x = 0 \) (DCM). \( 0 \leq x_j(n) \leq b \) and \( s_j(n) = -1 \) are required.

\[
x_j(n+1) = 0, \quad s_j(n+1) = 1
\]  

(7)

Type 5: \( x_j \) decreases without reaching \( x = 0 \) (CCM). \( b_j^{-1} > 1 \), \( b < x_j(n) \) and \( s_j(n) = -1 \) are required.

This HRM is of the form \( x_j(n+1) = f_j(x_j(n), s_j(n)) \) and \( s_j(n+1) = g_j(x_j(n), s_j(n)) \), \( j = 1 \sim N \). We refer to \((f_j, g_j)\), \( f_j \) and \( s_j \) as \( j \)-th component, \( x_j \) component and \( s_j \) component of the HRM, respectively. The \( j \)-th component interacts with other components through the WTA-switching that determines \( s_j(n+1) \) in Types 2 and 5.

The HRM can have a variety of shapes depending on parameters and we firstly consider the parameter region \( a_j^{-1} > 1 \) and \( b_j^{-1} > 1 \). In this case, Type 1, Type 2, Type 4 and Type 5 are possible and \( f_j \) has shape as shown in Fig. 4(a): \( f_j \) consists of 4 branches \( B_1 \), \( B_2 \), \( B_3 \) and \( B_5 \) corresponding to Type 1, 2, 4 and 5, respectively. Such a map is referred to as Class A hereafter. For all \( x_j(n) \) there exist two branches and the hitting branch is determined by \( s_j(n) \) as defined in Eqs. (4) to (8): if \( s_j(n) = 1 \) then an orbit hits \( B_1 \) or \( B_2 \) and if \( s_j(n) = -1 \) then an orbit hits \( B_3 \) or \( B_5 \). It should be noted that \( x_j(n) \) can escape from a “stable” fixed point on \( B_2 \) in Fig. 4(a) to \( B_5 \) when \( x_j(n) \) is not the winner. The orbit in Fig. 4(a) corresponds to 3-SYN in Fig. 2(a) where \( a_1 = a_2 = a_3 = a \). If \( a_j = a \) then the shape of \( f_j \) is independent of \( j \). In this figure \( x_1 \) is the winner at point \( \alpha \) at time \( \tau = n \) whereas \( x_2 \) and \( x_3 \) are at points \( \beta \) and \( \beta \), respectively. Since \( x_1(n) \) is the winner, the orbit hits \( B_2 \) corresponding to

\[
x_j(n+1) = x_j(n) - b
\]

\[
s_j(n+1) = \begin{cases} 
1 & \text{if } x_j(n+1) \text{ wins} \\
-1 & \text{otherwise}
\end{cases}
\]  

(8)
Type 2 trajectory. For two successive periods the trajectory of $x_1$ is Type 5 and the orbit hits $B_2$ from point $\beta$ and $\gamma$. The orbits of $x_2$ and $x_3$ repeat the same behavior as $x_1$ with delay 1 and 2, respectively. Figure 4(a') shows corresponding 3D plot that connects three points in $x_1$-$x_2$-$x_3$ space: $(\alpha, \gamma, \beta)$, $(\beta, \alpha, \gamma)$ and $(\gamma, \beta, \alpha)$. Here we note that the orbit of 3-SYN hits $B_2$ and $B_3$ having slopes $-p_j$ and 1, respectively. Since $0 < p_j = b/a_j < 1$ the the branch $B_2$ has contracting slope hence this 3-SYN is stable. As $a_j^{-1}$ increases, slope of $B_2$ becomes steep and the 3-SYN tends to be unstable. In general, we have Proposition 1 for $N \geq 2$.

**Proposition 1:** We assume existence of N-SYN in CCM. The N-SYN is stable if $b/a_j < 1$ for all $j$ and is unstable if $b/a_j > 1$ for some $j$.

Here we consider existence condition of N-SYN in CCM. The condition for $a_j = a$ is derived in [8] through elementary geometric discussion and that discussion is extended easily for $a_j \neq a$. Figure 5 shows an orbit of j-th converter for $\tau_0 < \tau < \tau_1$. It starts from the threshold $x_j = 1$ at time $\tau_0$, decreases until it becomes winner at time $\tau_a$, and increases until it hits the threshold $x_j = 1$ at time $\tau_1$ where $\tau_1 - \tau_0 = N$. Let $a_j \neq a$ and let $A_j = x_j(\tau_a) = 1 - N/((a_j^{-1} + b^{-1})$. Let $D_j = x_j(\tau_a - 1) = A_j + b$ for $b < a_j$ and let $D_j = x_j(\tau_a + 1) = A_j + a_j$ for $b \geq a_j$. $A_j$ is the minimum value of $x_j(n)$ and $D_j$ is the second minimum value of $x_j(n)$ as shown in Fig. 5. If $0 < A_j < 1$ and $A_j < D_j$ ($i \neq j$) are satisfied for all $j$ then each converter can be winner once for $\tau_0 \leq \tau < \tau_1$ and an N-SYN exists. We can conclude this discussion as the following.

**Proposition 2:** For $a_j \neq a$, N-SYN exists in CCM if $0 < A_j < D_j$ is satisfied for all $j \neq i$. For $a_j = a$, N-SYN exists in CCM if $1 > N/(a^{-1} + b^{-1})$.

If some N-SYN exists and $a_j^{-1}$ exceeds $b^{-1}$ then the N-SYN loses its stability. The orbit in Fig. 4(b) corresponds to 3-SYN in Fig. 2(b) where the branch $B_2$ has expanding slope, $p_j > 1$, hence the 3-SYN is unstable. We cannot observe this unstable 3-SYN but chaotic behavior as shown in Figs. 4(c) and (c'). These chaotic orbits hit branches $B_1$, $B_2$ and $B_3$ having slopes $1$, $-p_j$ and 1, respectively. Since $p_j > 1$, the orbits are expanding for three components $x_1$ to $x_3$. Chaotic orbits having two or more expanding directions are usually referred to as hyperchaos hence Fig. 4(c') shows hyperchaos. More detailed discussion on hyperchaos can be found in [23]. Noting that $x_j$ is unstable (expanding) if $p_j > 1$ and $x_j > 0$, we can say the following.

**Proposition 2:** The PDC exhibits chaos if $p_j = b/a_j > 1$ and $x_j > 0$ for some $j$. The PDC exhibits hyperchaos if $p_j = b/a_j > 1$ and $x_j > 0$ for two or more $j$.

As parameters vary, the HRM can have a variety of shapes. Shapes of $x_j$ component $f_j$ are classified into five classes:

Class A: If $a_j^{-1} > 1$ and $b^{-1} > 1$ then Types 1, 2, 4 and 5 are possible and $f_j$ consists of 4 branches $B_1$, $B_2$, $B_4$ and $B_5$ as shown in Fig. 4.

Class B: If $a_j^{-1} < 1$ and $b^{-1} > 1$ then Types 2, 4 and 5 are possible and $f_j$ consists of 3 branches $B_2$, $B_4$ and $B_5$.

Class C: If $a_j^{-1} > 1$ and $b^{-1} < 1$ then Types 1, 2, 3 and 4 are possible and $f_j$ consists of 4 branches $B_1$, $B_2$, $B_3$ and $B_4$.

Class D: If $a_j^{-1} < 1$, $b^{-1} < 1$ and $a_j^{-1} + b^{-1} > 1$ then Types 2, 3 and 4 are possible and $f_j$ consists of 3 branches $B_2$, $B_3$ and $B_4$.

Class E: If $a_j^{-1} + b^{-1} < 1$ then Types 3 and 4 are possible and $f_j$ consists of $B_3$ and $B_4$. However, all the trajectories are to be Type 3 (no Type 4) for $\tau > 1$ and it is sufficient to consider $f_j$ for $B_3$.

Figure 6 shows shapes of $f_j$ in Classes B to E. They can be calculated using Eqs. (4) to (8). The HRM is given by combination of $(f_j, g_j)$ for $j = 1$–$N$ selected from either of two groipes: [Class A, Class B] for $b^{-1} > 1$ and [Class C, Class D, Class E] for $b^{-1} < 1$. Figure 7 shows typical waveforms from Class B to E in the case $a_j = a$. 

![Fig.5](Image338x206 to 421x287) 

**Fig. 5** Existence of N-SYN in CCM.

![Fig.6](Image338x97 to 421x177) 

**Fig. 6** Typical Shapes of HRM in its $x_1$ component $f_1$. Class B: $(a_j^{-1}, b^{-1}) = (0.8, 3.3)$; Class C: $(a_j^{-1}, b^{-1}) = (1.5, 0.8)$; Class D: $(a_j^{-1}, b^{-1}) = (0.9, 0.8)$; Class E: $(a_j^{-1}, b^{-1}) = (0.5, 0.4)$. 

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4. Typical Bifurcation Phenomena

It is extremely cumbersome to investigate bifurcation phenomena in all possible combination of \((f_j, g_j)\) and therefore we consider three basic one-parameter bifurcation phenomena for \(N = 3\) in this section.

4.1 Bifurcation in CCM for \(a_j = a\)

Here we consider a bifurcation in the case \(a_1 = a_2 = a_3 = a\) and \(b^{-1} = 3.3\). In this case we have stable 3-SYN as shown in Fig. 4(a). As \(a^{-1}\) increases, slope \(-p_j = -b/a\) of \(B_2\) becomes steep, and the 3-SYN is changed to be unstable as shown in Proposition 1. The border of stability is given by \(a^{-1} = b^{-1}\) at which the slope of \(B_2\) changes from contracting \((p_j < 1)\) into expanding \((p_j > 1)\). In the bifurcation diagram of Fig. 8(a) we can see that stable 3-SYN is changed to be chaotic at the border \(a^{-1} = b^{-1} = 3.3\) where the 3-SYN lost stability. When \(a^{-1}\) exceeds \(b^{-1}\) the system exhibits hyperchaos in CCM as shown in Fig. 4(c).

Figure 8(b) shows ripple factor \(R_p\) of the dimensionless output current \(X\). In the stable region of \(a^{-1} < b^{-1}\) the \(R_p\) is small and has local minimum \(R_p = 0\) at \(2a^{-1} = b^{-1}\). As \(a^{-1}\) exceeds \(b^{-1}\), \(R_p\) jumps to be larger and exhibits smooth change with fluctuation. Note that \(R_p\) for hyperchaos is an approximation calculated for a sufficiently long time.

4.2 Bifurcation in CCM for \(a_j \neq a\)

Here we consider a bifurcation in the case \(a_1^{-1} = a_2^{-1} = 1.5\) and \(b^{-1} = 3.3\). Figure 9 shows typical phenomena changed from stable 3-SYN in Fig. 4(a) as \(a_3^{-1}\) varies. Note that \(a_3\) is proportional to the third dc input \(V_3\) and therefore the bifurcation for \(a_3^{-1}\) implies the bifurcation due to the imbalance of plural inputs. In the figure, the left column corresponds to distorted 3-SYN in Fig. 2(c).

In the HRM, \(f_1\) to \(f_3\) are all in Class A consisting of four branches. As \(a_3^{-1}\) varies, the shape of \(f_3\) varies whereas shapes of \(f_1\) and \(f_2\) are preserved. However, behavior of
$x_1$ and $x_2$ can change through the WTA-switching and the stable 3-SYN can be changed into chaotic behavior in the right column. Figures 10(a) to (c) show bifurcation diagrams where $x_1$, $x_2$ and $x_3$ components of the HRM are plotted as $a_3^{-1}$ increases. We can see that 3-SYN is distorted and is changed into chaos in CCM when $a_3^{-1}$ exceeds $b^{-1} = 3.3$. For $a_3^{-1} > b^{-1}$ the HRM exhibits chaotic behavior in CCM. Since the slope of $B_2$ is expanding for $x_3$ ($p_1 > 1$) whereas that is contracting for $x_1$ and $x_2$ ($p_1 = p_2 < 1$), only $x_3$ component is expanding in this chaotic behavior. Although $p_1$ and $p_2$ are not exceed 1, $x_1$ and $x_2$ behave chaotically affected by chaotic behavior of $x_3$ via the WTA-based switching. As far as this numerical simulation is concerned, contracting dynamics of $x_1$ and $x_2$ can not suppress chaotic dynamics of $x_3$. Figure 10(d) shows ripple factor $R_p$ that has characteristics similar to Fig. 8(b): $R_p$ is small and has local minimum at $a_3^{-1} = a_4^{-1} = a_5^{-1}$, jumps to be larger at $a_3^{-1} = b^{-1}$ and exhibits smooth variation with fluctuation for $a_3^{-1} > b^{-1}$.

4.3 Bifurcation Including DCM for $a_j \neq a$

Here we consider a bifurcation for $a_4^{-1}$ in the case $a_1^{-1} = a_2^{-1} = 1.2$ and $b^{-1} = 2.4$. Figure 11 left column shows the HRM and 3D plot corresponding to distorted 3-SYN in DCM in Fig. 2(d). In the figure, $f_1$ and $f_2$ are in Class A whereas $f_3$ is in Class B consisting of three branches $B_2$, $B_4$ and $B_5$. The orbit of $x_3$ hits branch $B_4$ having zero-slope that causes superstability in DCM whereas $x_1$ and $x_2$ do not hit $B_4$ and are in CCM. In the 3D plot, a point $x(n)$ is marked if $x_3$ hits $B_4$ ($x_3(n) = 0$): it is a signal of DCM and is superstable. As $a_3^{-1}$ increases, this 3-SYN is changed into complicated phenomena as shown in Fig. 11 right: $x_1$ to $x_3$ all hit $B_4$ and the HRM exhibits complicated superstable behavior. As far as basic numerical simulation is concerned, it is hard to judge whether such behavior is periodic or non-periodic. Figures 12(a) to (c) show bifurcation diagrams as $a_3^{-1}$ increases. We can see that 3-SYN in DCM is changed into 3-SYN in CCM and then to chaos in CCM. The border between stable 3-SYN and chaos in CCM is $a_3^{-1} = b^{-1} = 3.3$ and transition scenario through this border is similar to that in Figs. 8 and 10. As $a_3^{-1}$ increases further the chaos in CCM is changed into a variety of superstable phenomena in DCM. They are superstable for the initial state, however, they can be sensitive to parameters as suggested in the figure. Although such superstable behavior must be periodic in single converters [22], analysis of “complicated superstable behavior” in PDCs is in the future problems. Figure 12(d) shows ripple diagram. In 3-SYN and chaotic phases in CCM this diagram shows similar characteristics to Fig. 10: $R_p$ passes through one local minimum, increases and jumps to chaotic phase at $a_3^{-1} = b^{-1}$. For $a_3^{-1} > b^{-1}$, $R_p$ has large value with
fluctuation. Especially, $R_f$ in DCM is unnecessarily larger than $R_p$ of chaos in CCM in Figs. 8 and 10.

5. Conclusions

Typical bifurcation phenomena of multiple-input PDC are studied in this paper. In the analysis we have adopted PWC modeling that is well suited for precise calculations. Introducing the HRM of continuous and binary variables, we have investigated typical bifurcation phenomena. As parameters vary, N-SYN is changed into a variety of periodic/chaotic phenomena. Ripple characteristic is also investigated. These results provide basic information to design PDC with desired current sharing operation and to approach complicated bifurcation phenomena in higher dimensional systems.

It should be noted that experimental confirmation of typical phenomena for $a_j = a$ can be found in [24]: we have observed 3-SYN in CCM for $(a^{-1}, b^{-1}) = (1.6, 4.9)$, 3-SYN in DCM for $(a^{-1}, b^{-1}) = (1.4, 0.7)$ and chaotic orbit for $(a^{-1}, b^{-1}) = (3.5, 1.7)$. Experimental confirmation for $a_j \neq a$ is possible by replacing the single-input of the test circuit in [24] with multiple-input.

Future problems include detailed analysis of typical bifurcation phenomena, analysis of system performance such as ripple factor and efficiency, comparison of the PWC model with practical model as $RC$ decreases, and design of practical circuit.

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References


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