Resonance Phenomenon of Interspike Intervals From a Spiking Oscillator With Two Periodic Inputs

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Abstract—We consider a spiking oscillator to which two periodic and synchronized inputs can be applied. As either input is applied, typical phenomena from the oscillator are phase locking and quasi-periodic behaviors. Applying both inputs and adjusting their parameters appropriately, the system behavior approaches a resonance point characterized by: 1) The state is quasi-periodic; 2) The output pulse-train is periodic; and 3) They are not synchronized with the periodic inputs. Generating such resonance phenomena is impossible in the single input case. We present a simple implementation method of the spiking oscillator and verify the typical phenomena in the laboratory.

Index Terms—Interspike interval, quasi-periodicity, resonance, return map, spiking oscillator.

I. INTRODUCTION

IN THIS paper, we consider an oscillator to which multiple inputs can be applied. The multiple-inputs system would exhibit more interesting phenomena than single input systems. As a fundamental problem, it is important to recognize proper phenomena in the multiple-inputs system. It is also important to recognize basic functions of each input. If these functions can be fused efficiently in the multiple-inputs system, we may obtain more sophisticated functions than the single input systems. They will be fundamentals to develop multiple-inputs nonlinear systems, e.g., artificial neural networks, communications systems, and so on. Note that since the superposition theorem cannot be applied to the nonlinear systems, the general analysis will be hard. Hence, we focus on a simple but important system and consider proper phenomena to the multiple-inputs case.

We consider a spiking oscillator (SOC) to which two periodic and synchronized inputs $S(t)$ and $B(t)$ can be applied (see Fig. 1). The state $v$ is charged up according to the stimulation input $S(t)$. When the state $v$ reaches a threshold $V_T$, it is reset to a base level $B(t)$, and the SOC outputs a pulse. Repeating this manner, the SOC outputs a pulse-train $Y$. Note that, in our SOC, the inputs $S(t)$ and $B(t)$ are applied to different parts of the SOC. If either input is applied, typical phenomena from the SOC are phase locking and quasi-periodic behaviors as are often observed in forced oscillators [1]. Applying both inputs, the SOC exhibits a kind of resonance phenomena. In order to characterize the phenomena, we consider the density of the pulse intervals, interspike interval density (IID). In the quasi-periodic case, the IID is continuous. Adjusting the parameters of the inputs appropriately, the system behavior approaches to a resonance point which is characterized by: 1) the state $v$ is quasi-periodic; 2) the output $Y$ is periodic; and 3) they are not synchronized with the periodic inputs $S(t)$ and $B(t)$. That is, the IID approaches to a $\delta$-function. Using a mapping procedure, we clarify that generating the resonance phenomenon is impossible in the single-input case. Also, we present a simple implementation method of the SOC and demonstrate typical phenomena in the laboratory.

In the field of neural networks, the SOC can be regarded as an integrate-and-fire oscillator which is a simplified neuron model [1]–[4]. However, the SOC with multiple inputs has not been studied sufficiently. Our results may contribute to synthesis and analysis of pulse-coupled artificial neural networks with multiple inputs [5], [6]. Also, the implementation method may be developed into realization of such networks on chips [7], [8]. We have considered basic functions of the threshold input in [9], [10]. The preliminary results of this paper can be found in [11].

II. SPIKING OSCILLATOR

Fig. 1 shows the spiking oscillator (SOC) with two periodic inputs $S(t)$ and $B(t)$. For convenience, we refer to $S(t)$ and $B(t)$ as the stimulation input$^1$ and the base input, respectively. If the state $v$ is below the threshold $V_T > 0$, the dynamics are described by

$$C \frac{dv}{dt} = I_0 + S(t) \quad \text{for } v < V_T$$

$$I_0 > 0 \quad S(t) = K_S \sin \left( \frac{2\pi t}{T} \right). \quad (1)$$

The state $v$ is charged up by the current $I_0 + S(t)$. If the state $v$ reaches the threshold $V_T$, the monostable multivibrator (MM)

1$^1$I0 + S(t) and Y might correspond to the membrane potential, the stimuli and the spike-train of some neuron model, respectively [6], [12].
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Fig. 2. Basic dynamics of the spiking oscillator.

outputs an instantaneous pulse that closes the switch, and the state \( v \) is reset to the base level voltage \( B(t) \)

\[
v(t^+) = B(t^+), \quad \text{if } v(t) = V_T
\]

\[
B(t) = K_B \sin \left( \frac{2\pi}{T} (t - \Phi_B) \right), \quad |K_B| < V_T. \quad (2)
\]

Note that the inputs \( S(t) \) and \( B(t) \) have the same period \( T \), are synchronized to each other, and are applied to the different parts of the SOC. Repeating the switchings, the SOC outputs a pulse-train

\[
Y(t^+) = \begin{cases} 
    0, & \text{if } v(t) < V_T \\
    E, & \text{if } v(t) = V_T.
\end{cases} \quad (3)
\]

Using the dimensionless variables and parameters

\[
\tau = \frac{t}{T}, \quad x = \frac{v}{V_T}, \quad y = \frac{Y}{E}, \quad s_0 = \frac{I_0 T}{CV_T}, \quad k_s = \frac{K_s T}{CV_T}, \quad k_b = \frac{K_B}{V_T}, \quad \phi \in \left[ 0, \frac{2\pi}{T} \right].
\]

Equations (1)–(3) are transformed into

\[
\begin{align*}
\dot{x} &= s_0 + s(\tau), \quad \text{for } x(\tau) < 1 \\
\tau(t^+) &= b(\tau^+), \quad \text{if } x(\tau) = 1 \\
y(\tau^+) &= \begin{cases} 
    0, & \text{if } x(\tau) < 1 \\
    1, & \text{if } x(\tau) = 1
\end{cases} \\
s(\tau) &= k_s \sin(2\pi \tau) \\
b(\tau) &= k_b \sin(2\pi (\tau - \phi))
\end{align*}
\]

where \( \dot{x} \equiv dx/d\tau \). In order to simplify the analysis, we focus on the following parameters condition:

\[
|k_s| + 2\pi |k_b| < s_0 < 1, \quad |k_b| < 1. \quad (6)
\]

Fig. 2 shows the basic dynamics of the SOC. Let \( \tau_n \) be the \( n \)th pulse position (switching moment), and let \( \Delta \tau_n \equiv \tau_{n+1} - \tau_n \) be the \( n \)th interspike interval. Also, let \( x_m \equiv x(m + \phi_\theta) \) be the state sampled at time \( m + \phi_\theta \), where \( m \) is a nonnegative integer. Fig. 3 shows typical phenomena from the SOC. Since the inputs \( s(\tau) \) and \( b(\tau) \) are period 1, we introduce the pulse phase \( \theta_n \equiv \tau_n \pmod{1} \). \( \alpha(\theta) \), \( \beta(\Delta \tau) \) and \( \gamma(x) \) denote the densities of \( \theta_n \), \( \Delta \tau_n \) and \( x_m \), and are called phase density, interspike interval density (IID) and state density, respectively. In Fig. 3, they are calculated using return maps discussed in the next section.

We can summarize typical phenomena in Table I. Applying both inputs \( s(\tau) \) and \( b(\tau) \) and adjusting their parameters appropriately, the phenomena approaches to (d): the IID approaches to the \( \delta \)-function as in Fig. 3(d). Then, the SOC exhibits a kind of resonance phenomenon of the IID, where (d) can be considered as a resonance point. In the following sections, we explicate the phenomena and show that generating the resonance phenomenon is impossible in the single input case.
TABLE I

<table>
<thead>
<tr>
<th>Input</th>
<th>State $x$</th>
<th>Output pulse-train $y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single input $s(t)$ or $b(t)$</td>
<td>Periodic</td>
<td>Periodic</td>
</tr>
<tr>
<td>Both inputs $s(t)$ and $b(t)$</td>
<td>Quasi-periodic (a)(b)</td>
<td>Quasi-periodic (c)</td>
</tr>
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</table>

III. PHASE MAP AND STATE MAP

First, let us consider the dynamics of the output pulse-train $y$. Integrating (5), the state $x$ for $\tau_n \leq \tau < \tau_{n+1}$ is given by

$$x(\tau) = b(\tau_n) + \int_{\tau_n}^{\tau} (s_0 + s(\tau')) \, d\tau' + k_s \sin(2\pi(\tau_n - \phi_k)) + \int_{\tau_n}^{\tau} (s_0 + k_s \sin(2\pi \tau')) \, d\tau'$$

(7)

(see Fig. 2). Substituting $(\tau, x) = (\tau_{n+1}, 1)$ into (7), the relationship between $\tau_n$ and $\tau_{n+1}$ is given by

$$H(\tau_{n+1}) = H(\tau_n) + \frac{1}{s_0} (1 - k_s \sin(2\pi(\tau_n - \phi_k)))$$

(8)

where

$$H(\tau) = \frac{1}{s_0} \int_{0}^{\tau} (s_0 + k_s \sin(2\pi \tau')) \, d\tau' = \tau + \frac{k_s}{2s_0} (1 - \cos(2\pi \tau)).$$

(9)

Then, we have the following theorem.

**Theorem 1:** The parameters condition (6) guarantees that $H$ is continuous, monotone-increasing, and thus invertible. Also, $H(0) = 0$ and $H(\tau + 1) = H(\tau) + 1$. Then, (8) can be solved for $\tau_{n+1}$

$$\tau_{n+1} = f(\tau_n)$$

$$\equiv H^{-1} \left( H(\tau_n) + \frac{1}{s_0} (1 - k_s \sin(2\pi(\tau_n - \phi_k))) \right),$$

(10)

where $f$ is continuous, monotone-increasing and $f(\tau + 1) = f(\tau) + 1$.

**Proof:** From (9), $H$ is continuous. Differentiating $H$, we obtain

$$\frac{dH(\tau)}{d\tau} = 1 + \frac{k_s}{s_0} \sin(2\pi \tau).$$

The first inequality in (6) guarantees $dH/d\tau > 0$, and thus $H$ is monotone-increasing and invertible. Substituting $\tau + 1$ into (9), we can check $H(\tau + 1) = H(\tau) + 1$. Since $H$ and $H^{-1}$ are continuous, $f$ is continuous. Differentiating $f$, we obtain

$$\frac{df(\tau)}{d\tau} = \frac{dH^{-1}(K)}{dK} \frac{dK(\tau)}{d\tau}$$

$$K(\tau) \equiv H(\tau) - \frac{1}{s_0} (1 - k_s \sin(2\pi(\tau - \phi_k)))$$

and

$$\frac{dK(\tau)}{d\tau} = 1 + \frac{k_s}{s_0} \sin(2\pi \tau) + \frac{2\pi k_0}{s_0} \cos(2\pi(\tau - \phi_k)).$$

The first inequality in (6) guarantees $dK/d\tau > 0$. Then, $f$ is monotone-increasing since $dH^{-1}/dK > 0$. Since $H(\tau + 1) = H(\tau) + 1$, we have $f(\tau + 1) = f(\tau) + 1$. Note that the second inequality in (6) prohibits the situation $b(\tau) > 1$ and prohibits the existence of stable fixed point(s) of $f$.

Q.E.D.

Since the pulse position $\tau_n$ is governed by $f$, we refer to $f$ as the pulse position map. Fig. 4 shows an example of $f$. Since $f(\tau + 1) = f(\tau) + 1$, the system dynamics can be analyzed by the following return map

$$\theta_{n+1} = F(\theta_n) \equiv f(\theta_n) \quad (\text{mod } 1), \quad F : [0, 1] \rightarrow [0, 1]$$

(11)

where $\theta_n = \tau_n \text{(mod } 1)$. We refer to this map as the phase map. In Fig. 4, the phase map $F$ is shown. The function $H$ represents effects of the stimulation input $s(\tau)$ on the phase map $F$. From Theorem 1, $F$ is a circle map (circle homeomorphism). In order to consider the pulse-train dynamics, we recall some basic results on the circle map [13]–[15].

**Basic Properties of the Circle Map:**

- The rotation number $\rho$ is defined by

$$\rho \equiv \lim_{n \to \infty} \frac{1}{n} (f^n(\tau_1) - \tau_1).$$

(12)

Then, $\rho$ exists and is independent of the initial state $\tau_1$.

- The return map $F$ exhibits periodic orbits if and only if $\rho$ is rational. In this case the output pulse-train $y(\tau)$ synchronizes with the inputs $s(\tau)$ and $b(\tau)$.

- The return map $F$ exhibits quasi-periodic orbits if and only if $\rho$ is irrational. In this case $F$ has a unique invariant density, i.e., $F$ is ergodic. Then, the output pulse-train $y(\tau)$ does not synchronize with the inputs $s(\tau)$ and $b(\tau)$.

- If $\rho$ is irrational, $F$ is topologically conjugate to the rotation map $R(\theta) \equiv \theta + \rho \text{(mod } 1)$.

That is, there exists a continuous, monotone-increasing and one-to-one function (homeomorphism) $h : [0, 1) \rightarrow [0, 1)$ such that

$$F = h^{-1} \circ R \circ h,$$

(13)
In this case, the invariant density of $F$ (phase density) is given by
\[ \alpha(\theta) = \frac{d\theta(\theta)}{d\theta}. \] (14)

In the following, we focus on the irrational $\rho$. Using the phase density $\alpha(\theta)$, the distribution function $B_f(\Delta \tau)$ of the interspike interval $\Delta \tau_n$ is given by
\[ B_f(\Delta \tau) = \int_{\theta \equiv \theta \leq \Delta \tau} \alpha(\theta) d\theta. \] (15)

Then, the IID $\beta(\Delta \tau')$ is such that
\[ \int_{\Delta \tau' \leq \Delta \tau} \beta(\Delta \tau') d\Delta \tau' = B_f(\Delta \tau), \] (16)

The mean interspike interval is given by the rotation number $\rho$, i.e.,
\[ \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \Delta \tau_i = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} (f(\tau_i) - \tau_i) = \rho. \] (17)

Next, let us consider the dynamics of the sampled state $x_m = x(m + \phi_0)$, $m = 0, 1, \ldots$. The first equation in the condition (6) guarantees that the state $x$ starting from $x(\phi_0) \in [0, 1]$ exhibits no switching [resetting to $\theta(\tau)$] or one switching during $\phi_0 \leq \tau < 1 + \phi_0$. Then, from a simple calculation, the sampled state $x_m$ is described by the following return map:
\[ x_{m+1} = G(x_m), \quad G: [0, 1] \to [0, 1] \] (18)

We refer to this map as the state map. In Fig. 5, the state map $G$ is shown. Note that for the irrational rotation number $\rho$, not only the phase map $F$ but also the state map $G$ are ergodic, and thus the state $x_m$ is quasi-periodic.

In Fig. 3, the phase density $\alpha(\theta)$, $\beta(\Delta \tau)$ (IID) and the state density $\gamma(x)$ are given by the histograms of $\theta_n$, $\Delta \tau_n$ and $x_m$ which are calculated by iterating $F$ and $G$. We choose the iteration number $N = 10000$, since the standard deviation of the histograms change within $\pm 0.1\%$ for $N > 10000$. Note that the ergodicity means that the histograms approach to the densities independently of the initial states as $N \to \infty$.

**IV. Resonance Phenomena**

First, let us consider the case where only the base input $u(\tau)$ is applied ($j_{k_b} = 0$). In this case, the phase map $F$ is given by
\[ F(\theta_n) = \theta_n + \frac{1}{s_0} (1 - b(\theta_n))(\text{mod} \ 1). \] (19)

Note that the shape of the phase map $F$ is determined by the shape of the base input $u(\tau)$, that is, the base input can realize rich pulse-train dynamics [9], [10]. The state map $G$ is given by
\[ G(x_m) \equiv \begin{cases} x_m + s_0, & \text{for } 0 \leq x_m < 1 - s_0, \\ x_m + s_0 - 1 + b \left( H^{-1} \left( \frac{1 - x_m}{s_0} \right) + H(\phi_0) \right), & \text{for } 1 - s_0 \leq x_m \leq 1. \end{cases} \] (20)

Note that the shape of the phase map $F$ is shown in the shape of the base input $u(\tau)$, that is, the base input can realize rich pulse-train dynamics [9], [10]. The state map $G$ is given by
\[ G(x_m) \equiv \begin{cases} x_m + s_0, & \text{for } 0 \leq x_m < 1 - s_0, \\ x_m + s_0 - 1 + b \left( \frac{1 - x_m}{s_0} + \phi_0 \right), & \text{for } 1 - s_0 \leq x_m < 1. \end{cases} \] (21)

Fig. 5(a) shows the phase map $F$ and the state map $G$. If the rotation number $\rho$ is irrational, $F$ and $G$ exhibit quasi-periodic orbits. Then, the pulse phase $\theta_n$ and the state $x_m$ are quasi-periodic as shown in Fig. 3(a). Noting the phase density $\alpha(\theta)$ is continuous and $f(\theta - \theta)$ is not constant, Equations (15) and (16) guarantee that the IID is continuous. Then, the output pulse-train $y(\tau)$ is quasi-periodic.

Second, let us consider the case where only the stimulation input $s(\tau)$ is applied ($j_{k_b} = 0$). In this case, the phase map $F$ is given by
\[ F(\theta_n) = H^{-1} \circ R \circ H, \quad R(\theta) = \theta + \rho \] (21)

where $H$ corresponds to the homeomorphism $h$ in (13), and the rotation number is $\rho = 1/s_0$. That is, the stimulation input $s(\tau)$...
Fig. 6. Characteristics of the standard deviation $\sigma$ of the interspike interval density $\beta(\Delta \tau)$, $s_0 = 0.5 \sqrt{3}$, $k_s = 0.25$, $k_{\text{res}} = 0.037$, $\phi_{\text{res}} = 0.422$. The points (b), (c), and (d) correspond to (b), (c) and (d) of Fig. 5 (and Fig. 3), respectively.

has a transformation function for the domain of the phase map $F$. The state map $G$ is given by

$$G(x_m) = \begin{cases} x_m + s_0, & \text{for } 0 \leq x_m < 1 - s_0 \\ x_m + s_0 - 1, & \text{for } 1 - s_0 \leq x_m < 1. \end{cases} \quad (22)$$

Fig. 5(b) shows the phase map $F$ and the state map $G$. If $\rho$ is irrational, $F$ and $G$ exhibit quasi-periodic orbits as shown in Fig. 3(b). Note that rational $\rho$ is measure zero for $s_0$. Then, from the same consideration as the base input case, the state $x$ and the output pulse-train $y$ are quasi-periodic.

Third, let us apply both inputs $s(\tau)$ and $b(\tau)$. As $k_b$ increases, we can observe the following phenomena:

1. $k_b = 0$ [Figs. 3(b) and 5(b)].

The IID has the continuous distribution and its standard deviation is $\sigma \simeq 0.03$. The output pulse-train $y$ and the state $x$ are quasi-periodic because the IID and the state density $\gamma(x)$ are continuous.

2. $k_b = 0.019$ [Figs. 3(c) and 5(c)].

The IID is distributed on the narrower region and the standard deviation is $\sigma \simeq 0.02$. The output pulse-train $y$ and the state $x$ are quasi-periodic similarly to (b).

3. $k_b = 0.037$ [Figs. 3(d) and 5(d)].

The IID is a $\delta$-function and the standard deviation is $\sigma = 0$. Then, the output pulse-train $y$ is periodic with a constant interspike interval. However, the output $y$ is not synchronized with the inputs $s(\tau)$ and $b(\tau)$ because the phase density $\phi(\theta)$ is continuous. The state $x$ is quasi-periodic because the state density $\gamma(x)$ is continuous.

Fig. 6 shows the characteristics of the standard deviation $\sigma$ of the IID for the parameters $k_b$ and $\phi_b$, where the other parameters $s_0$ and $k_s$ are fixed. The points denoted by (b), (c), and (d) correspond to the above phenomena (b), (c), and (d), respectively. At the point d), the parameters are $(k_b, \phi_b) = (k_{\text{res}}, \phi_{\text{res}})$, and the standard deviation $\sigma$ is the minimum. Then, we say that the SOC exhibits a resonance phenomenon of interspike intervals and refer to $(k_{\text{res}}, \phi_{\text{res}})$ as the resonance point. In this paper, we use the term resonance in a wide sense: as a response characteristics of some physical quantity has an extreme with respect to an input parameter. For example, the stochastic resonance [16] represents the following phenomenon. In some nonlinear oscillators with a noisy input, the signal-to-noise ratio is maximized at a certain noise intensity. Roughly speaking, the mechanism of our resonance phenomenon can be explained as the following (see Fig. 2). During a higher part of the stimulation input $s(\tau)$, the state $x$ increases more rapidly and the interspike interval $\Delta \tau(n)$ tends to be shorter. If the SOC fires and the state $x$ is reset to a lower value of the base $b(\tau)$, the interspike interval $\Delta \tau(n)$ tends to be longer. At the resonance point, these effects cancel each other and the interspike interval $\Delta \tau(n)$ becomes a constant. For the resonance point, we have the following theorem.

**Theorem 2:** For a given $s_0$, the pulse position map $f$ becomes the linear map

$$\tau_{n+1} = f(\tau_n) = \tau_n + \frac{1}{s_0} \quad (23)$$

if and only if the parameters of the inputs $s(\tau)$ and $b(\tau)$ satisfy

$$k_b = k_{\text{res}} \equiv -k_s \frac{\pi}{\sin \left( \frac{\pi}{s_0} \right)}, \quad \phi_b = \phi_{\text{res}} \equiv 1 - \frac{1}{2s_0}. \quad (24)$$

**Proof:** “If” part: Substituting (24) into (10), we obtain (23).

“Only if” part: Substituting (23) into (8), we obtain

$$H \left( \tau_n + \frac{1}{s_0} \right) = H(\tau_n) + \frac{1}{s_0} \left( 1 - k_{\text{res}} \sin(2\pi(\tau_n - \phi_{\text{res}})) \right). \quad (25)$$

Comparing the parameters on both sides, we obtain (24). Q.E.D.

Based on this theorem, we explain the phenomenon (d).

- From (23), the interspike interval is given by

$$\Delta \tau_n = \tau_{n+1} - \tau_n = \frac{1}{s_0}, \quad \text{for all } \tau_n. \quad (26)$$

Hence, the pulse-train $y(\tau)$ is periodic with the constant interspike interval $1/s_0$. The IID becomes the $\delta$-function, $\beta(\Delta \tau) = \delta(\Delta \tau - 1/s_0)$, and its standard deviation is $\sigma = 0$.

- If the rotation number $\rho$ is irrational, the phase map $F$ and the state map $G$ are ergodic; the pulse phase $\theta_n$ and the sampled state $x_{\tau_n}$ are quasi-periodic. Note again that rational $\rho$ is measure zero. Then, the periodic pulse-train $y(\tau)$ does not synchronize with the periodic inputs $s(\tau)$ and $b(\tau)$.

Equation (26) means that $\Delta \tau_{\text{res}} = 1/s_0$ is satisfied independently of the first pulse position $\tau_1$. Since $\tau_1$ is given by the initial state $x(0)$ such that $1 = x(0) + \int_0^\tau (s(\tau) + s(\tau)) \, d\tau$ (see Fig. 2), the periodic pulse-train is stable for the initial state. Also, Fig. 6 suggests that $\sigma$ changes continuously with respect to the change of $k_b$ and $\phi_b$. This will be a reason for the fact that we can observe the phenomenon (d) in the laboratory as the
parameters values approach to the resonance point (see the next section).

Remark 1: In the single input case, \(k_s = 0\) or \(k_0 = 0\), (24) is not satisfied and the pulse position map \(f\) is not linear. Then, the interspike interval \(\Delta T\) is not constant and the IID has a continuous distribution. Hence, generating the resonance phenomenon is impossible in the single input case for almost all \(\mathcal{S}_0\).

Remark 2: Let us consider the case where the inputs \(s(\tau)\) and \(b(\tau)\) are not sinusoidal. If the inputs are continuous, periodic as \(s(\tau + 1) = s(\tau)\) and \(b(\tau + 1) = b(\tau)\), and satisfy

\[
s(\tau) + s_0 > \max_{\tau} \frac{db(\tau)}{d\tau} \tag{27}
\]

then \(H\) in (9) is continuous and monotone-increasing. In addition, if

\[
b(\tau) = s_0 \left(H(\tau) - H\left(\tau + \frac{1}{s_0}\right)\right) + 1 \tag{28}
\]

the pulse position map \(f\) becomes linear as in (23). Hence, the SOC would exhibit similar resonance phenomena if such inputs are parameterized. The detailed analysis is in future problems.

V. IMPLEMENTATION

In Fig. 7(a), we present a simple implementation method of the SOC using the programmable unijunction transistor (PUT). Fig. 7(b) shows the basic characteristics of the PUT (N13T1) [17]. Basically, it has three branches. The left \((i < i_T)\), the middle \((i_T < i < i_B)\) and the right \((i_B < i)\) branches. If \(v < B_P(t) + c_B\), the current \(i\) is on the left branch and thus \(i = 0\), the PUT is closed. If we increase \(v\) until \(V_{TP} + c_T\), the PUT remains closed. However, if \(v\) reaches the upper threshold \(V_{TP} + c_T\), the current \(i\) jumps to the right branch and \(i\) has large values, the PUT is opened. This remains until \(v\) reaches the lower threshold \(B_P(t) + c_B\). The parameters \(c_B \approx 0.5\) [V], \(c_T \approx 0.5\) [V] and \(i_T \approx 0\) [mA] are almost constant. The slope of the middle branch (and thus \(i_B\)) can be adjusted by \(R_p\). Adjusting \(R_p\) appropriately, the PUT operates in the same manner as the set of comparator, MM and controlled switch in Fig. 1. The base and the threshold levels are given by

\[
B(t) = B_P(t) + c_B, \quad V_T = V_{TP} + c_T. \tag{29}
\]

Then, the dynamics of Fig. 7(a) is described by (1) and (2). \(Y_P\) corresponds to the output pulse-train \(Y: Y_P = B(t)\) just after the switching moments and \(Y_P = Y_{TP}\) otherwise. Fig. 7(c) shows the input signals \(S(t)\) and \(B(t)\) generator. The signals \(-A\sin(2\pi t/T)\) and \(V_T\) are the voltage inputs to the generator.
The operational transconductance amplifiers (NJM13600) realize the current $I_0 + S(t)$ such that

$$I_0 = \frac{V_i}{R_S}, \quad S(t) = K_S \sin \left( \frac{2\pi}{T} t \right), \quad K_S = \frac{A}{R_S}. \quad (30)$$

The voltage signal $B_p(t)$ is given by

$$B_p(t) = K_B \sin \left( \frac{2\pi}{T} (t - \Phi) \right), \quad K_B = \frac{1}{2} AK$$

$$\Phi_B = \tan^{-1} \frac{2\gamma}{1 - \gamma^2}, \quad \gamma = \frac{2\pi}{T} Rp Cl. \quad (31)$$

The parameters $K_B$ and $\Phi_B$ can be adjusted by $K$ and $R_p$, respectively. By changing $R_p$, the phase $\Phi_B$ can be varied for $0 \leq \Phi_B \leq \pi$. Also, by giving negative $K$, the phase can be varied for $\pi \leq \Phi_B \leq 2\pi$. Note that the phase cannot be adjusted in the wide region by using only $R_p$ and $C_p$ [18]. Fig. 8 shows laboratory measurements, where (a)–(d) correspond to (a)–(d) in Figs. 3, 5, 6, and Table I.

VI. CONCLUSIONS

We have considered the spiking oscillator to which two synchronized periodic inputs can be applied. If either input is applied, the oscillator exhibits phase locking or quasi-periodic behaviors. Applying both inputs and adjusting their parameters appropriately, the system behavior approaches to the resonance point. The state is quasi-periodic and the output is periodic but not synchronized with the periodic inputs. Generating the resonance phenomenon is impossible in the single input case. We have clarified the parameters condition that causes the resonance phenomenon theoretically. We have presented the simple implementation method and have verified typical phenomena in the laboratory. Future problems include the following:

• analysis of the resonance phenomena and related bifurcation phenomena for various kinds of inputs;
• clarification of the relationship between the bifurcation phenomena and pulse codings;
• synthesis and analysis of pulse-coupled networks of spiking oscillators, and design of their simple implementation circuits [12].

REFERENCES


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