First, the 1-D condition \( f(z_1, 0) \neq 0, \ |z_1| \leq 1 \) is verified. Second, the condition \( PE_2(0) > PE_2'(0) \) with \( |z_1| = 1 \) or \( a_{11}'(1) \cdot T \cdot x = 119 - 518x + 416x^2 > 0 \ \forall x \) with \(-1 \leq x \leq 1\) is examined. The roots of the last polynomial are 0.3039 and 0.9413, so for \( x = 0.5 \) it is negative. Therefore, the corresponding 2-D system (1) is unstable. One can obtain that \( f(0.4 + 0.8i, -0.186 + 0.26 - 0.904 + 0.03i) = 0 \), which assures the above result.

Example 3: Consider [15, Example 2].

\[
f(z_1, z_2) = \begin{bmatrix} z_1 & z_2^2 & z_3^3 \\ 1 & 0.1 & 0.25 & 0.1 \\ 0.7 & 1.25 & 1.5 & 1.3 \\ -0.4 & -0.85 & -2 & 1.2 \\ -0.25 & 1.7 & -0.9 & 0.1 \\
\end{bmatrix} \begin{bmatrix} z_2 \\ z_3^2 \\ z_3 \\
\end{bmatrix}.
\]

(27)

The 1-D stability conditions are satisfied. By using \( PE_2(0) > PE_2'(0) \) one finds \(-1.4375 - 2.58 \cos \theta - 1.65 \cos 2\theta - 0.52 \cos 3\theta > 0 \ \forall \theta, 0 \leq \theta \leq 2\pi \), which directly fails for \( \theta = 0 \). Thus, this 2-D system is unstable. The same result can be found by examining the equivalent inequality \( a_{11}'(1) \cdot T \cdot x = 0.2125 - 1.02x - 3.3x^2 - 2.68x^3 > 0 \ \forall x \) with \(-1 \leq x \leq 1\).

IV. CONCLUSION

The minimal delay property gives useful necessary conditions for stability of a 1-D polynomial. This property can also supply us with useful necessary conditions for the stability of 2-D polynomials. A comparison with other necessary stability conditions is given via some useful necessary conditions for the stability of 2-D polynomials. A comparison with other necessary stability conditions is given via some useful necessary conditions for the stability of 2-D polynomials.

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Basic Synchronous Phenomena by Intermittently Coupled Capacitors

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Abstract—We propose a simple coupling method using intermittently coupled capacitors (ICC’s). We construct a coupling system by applying the ICC to two piecewise linear nonautonomous chaotic circuits. Then the ICC changes the two chaotic attractors into a coexisting state of chaos synchronization and periodic synchronization. The system exhibits one of them depending on the initial states. The system dynamics are reduced into a 3-D return map with one real and two binary variables and the occurrence of the coexisting phenomenon is guaranteed theoretically. Also, typical phenomena are confirmed in the laboratory.

II. CONCLUSION

The study of chaos synchronization is important not only as a basic nonlinear problem but also for new engineering applications, including artificial neural networks [1]–[3] and chaos-based communications [4], [5]. In the studies, there are two basic problems. First, the nonlinear system exhibits various kinds of synchronous phenomena, including phase synchronization [6], [7] and they should be classified

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I. INTRODUCTION

The study of chaos synchronization is important not only as a basic nonlinear problem but also for new engineering applications, including artificial neural networks [1]–[3] and chaos-based communications [4], [5]. In the studies, there are two basic problems. First, the nonlinear system exhibits various kinds of synchronous phenomena, including phase synchronization [6], [7] and they should be classified
clearly. Second, the synchronous phenomena should be recognized theoretically against their accompanying troublesome problems, e.g., riddled basins [8]. Since it is almost impossible to solve the problems generally, some simple coupling methods have been considered. For example, [9] and [10] consider an impulsive synchronization method that realizes master-slave in-phase synchronization of chaos by sudden changing of the state values to the master states. Also, an occasional linear connection (OLC) method [1], [2] realizes mutual synchronization of chaos.

In this paper, we propose a simple coupling method using impulsive switched capacitors (ICC’s) that short some capacitors periodically and instantaneously. The ICC method can be regarded as a mutually coupling version of the impulsive synchronization method and as a simplified version of the OLC. We construct a coupling system by applying the ICC to two piecewise linear nonautonomous circuits whose chaotic generation is guaranteed theoretically [2]. Then the ICC changes the two chaotic attractors into a coexisting state of chaos synchronization and periodic attractors1. The system exhibits one of them depending on the initial states. The system dynamics can be reduced into a three-dimensional (3-D) return map with one real and two binary variables. Using the map, we can give a theoretical evidence for the coexisting phenomenon. Such a coexistence phenomenon has been reported in [4]. However, the ICC is a novel coupling method and the mechanism of the coexistence phenomenon is not discussed sufficiently in [4]. In addition, this paper provides a simple implementation method of the coupled system and demonstrates typical phenomena.

II. INTERMITTENTLY COUPLED CAPACITORS

We consider a coupled system, as shown in Fig. 1, where two nonautonomous chaos generators [2] are coupled by a switch SW. First, we consider each chaos generator without the coupling (SW is open all the time)

\[ RC_i \frac{dv_i}{dt} = (RG - 1)v_i + H(v_i, t), \quad i = 1, 2 \quad (1) \]

1Reference [11] gives an experimental result on an occurrence of a periodic attractor when two chaotic circuits are coupled.

where \(-G\) is a linear negative conductor, \(Th(t)\) is an op-amp saturation voltage, and \(a^{-1} = R_a/(R_a + R_c)\). \(H(v, t)\) is a time-variant hysteresis that is switched from \(-E\) to \(E\) (respectively, from \(-E\) to \(0\)) if \(v\) reaches a periodic threshold \(Th(t)\) (respectively, \(-Th(t)\)) holding \(v\) constant. The threshold is controlled by a periodic switch \(S_c\) that is opened (respectively, closed) during the first half period (respectively, second half period). It is guaranteed theoretically that this circuit exhibits chaotic attractors, an example of which is shown in Fig. 2(a) [2].

The two chaotic circuits are coupled by the switch SW that is closed only at each period end \(t = nT, n = 0, 1, 2, \ldots\). At the switching moment, two capacitors’ voltages are equalized instantaneously

\[ v_1(t) = v_2(t) = \frac{1}{C_1 + C_2} (C_1 v_1(t_-) + C_2 v_2(t_-)), \quad \text{for } t = nT \quad (2) \]

where \(v_1(t_-)\) denotes the voltage just before the switching2. We refer to these switched capacitors as ICC’s. For simplicity, let \(C_1 = C_2 \equiv C\) hereafter. For \(t \neq nT\), the circuit dynamics are described by

\[
\begin{align*}
RC_1 \frac{dv_1}{dt} &= (RG - 1)v_1 + H(v_1, t), \\
RC_2 \frac{dv_2}{dt} &= (RG - 1)v_2 + H(v_2, t),
\end{align*}
\]

In Fig. 2, we can see that the ICC changes two chaotic attractors [Fig. 2(a)] into a coexisting state of chaos synchronization (Fig. 2(b)), a periodic attractor (Fig. 2(c)), and their symmetric ones. The system

2Such instantaneous switching is not correct in a physical sense: the energy continuity property is broken. However, this KVL-based approximation is accepted in switched capacitor techniques.
Fig. 2. Laboratory measurements. The time-domain waveforms are traced seven times. $R = 51$ [kΩ], $C_1 = C_2 = 10$ [nF], $G^{-1} = 30$ [kΩ], $E = 8$ [V], $R_a = 10$ [kΩ], $R_b = 51$ [kΩ], $T = 278$ [μsec] ($p = 0.11, \lambda = 0.15$). (a) Chaotic attractor from the unit circuit. (b) Chaos Synchronization. (c) Periodic attractor.

Fig. 3. Definition of basic map.

exhibits one of them depending on the initial states. Hereafter, we explain this phenomenon theoretically.

Using the following dimensionless variables and parameters:

$$\tau = \frac{t}{T}, \quad x_i = \frac{a}{E} v_i, \quad p = \frac{1}{a} (RG - 1), \quad \lambda = \frac{RC}{aT}$$

(2), (3) are transformed into

$$\begin{align*}
\lambda x_1 &= px_1 + y_1, \quad y_1 \equiv h(x_1, \tau), \\
\lambda x_2 &= px_2 + y_2, \quad y_2 \equiv h(x_2, \tau), \\
x_1(\tau) &= x_2(\tau) \equiv \frac{1}{2} (x_1(\tau^-) + x_2(\tau^-)), \quad \text{for } \tau \neq n
\end{align*}$$

(5)

where

$$x_i \equiv \left(\frac{d}{d\tau}\right)x_i, \quad h(x_i, \tau) \equiv \frac{1}{E} H(a/E x_i, T \tau), \quad \text{and} \quad th(\tau) \equiv \frac{1}{E} \Theta (T \tau).$$

First, we consider the unit chaos generator whose parameters are restricted as the following:

$$\begin{align*}
\left\{ (p, \lambda, a) \mid p > 0, \quad \frac{1}{2} < T_p < 1, \\
- \frac{1}{p} < -a < \left(-x_c - \frac{1}{p}\right) e^{\frac{2a}{p}} + \frac{1}{p} \right\}
\end{align*}$$

(6)

where $T_p \equiv (\lambda/p) \ln(1 + p/1 - p)$ and $x_c \equiv (1 - 1/p) e^{-(p/2)\lambda} + (1/p)$ are illustrated in Fig. 3. The trajectory started from $(x_i, y_i) =$
However, the switching time map is not convenient to analyze the ICC system, hence, we use the state map $F$. (see Fig. 4(b)). Since Condition (8) also guarantees $|x_n| < x_0$ for the attractor, $y_1(n + 1) = -y_n(n)$ is satisfied. Hence, the dynamics are described by alternative application of $f(x, 1)$ and $f(x, -1)$. Noting $f(f(x, -1), 1) = -f(-f(x, -1), -1) = F(F(x))$, we notice that (8) guarantees attractor existence of the original system. Differentiating $f$ in (7), we obtain $|dF/dx| > 1$ on $I_a$ except for the break points. In this case, [12] guarantees that $F$ on $I_a$ is ergodic and has a positive Lyapunov exponent: the system exhibits chaos.

Note that Chaos in Lemma 1 corresponds to the chaotic attractor of the original system for the initial states $x_i(0) \in I_a$ and $y_i(0) = -1$ and that its symmetric attractor exists for $x_i(0) \in I'_a$ and $y_i(0) = 1$, where $I'_a$ is symmetric to $I_a$.

Assuming (8), we consider the ICC system. That is, we consider the case where the ICC is applied to two chaos generators that exhibit two symmetric periodic attractors if $\alpha > 0$ and that its symmetric attractor exists for $x_i(0) \in I'_a$ and $y_i(0) = 1$, where $I'_a$ is symmetric to $I_a$.

Proof: Differentiating (7), we obtain $(d/d\bar{x})F' = 1/2(e^{\nu/3} + e^{\nu/3}) > 1$ for $|\bar{x}| < x_a$, and $(d/d\bar{x})F'_a = 1/2(e^{\nu/3} - e^{\nu/3}(1-\nu)) > 0$ for $x_a < |\bar{x}| < x_b$: $F'_a$ is monotone on $(-x_b, x_b)$. Hence, (10) guarantees that $F'_a(\bar{x}, 1, -1)$ has two symmetric stable fixed points, as shown in Fig. 5. Noting $F'_a(\bar{x}, 1, -1) = F'_a(\bar{x}, -1, 1)$, the 3-D return map can be simplified into

$$\bar{x}(n + 1) = F_a(\bar{x}(n), y_1(n), y_2(n)) \equiv \frac{1}{2}(f(\bar{x}(n), y_1(n)) + f(\bar{x}(n), y_2(n)))$$

$$y_1(n + 1) = g(\bar{x}(n), y_1(n)), \quad y_2(n + 1) = g(\bar{x}(n), y_2(n)).$$

If $y(0) = y_2(0)$ and $\bar{x}(0) \in I_a$, $F_a(\bar{x}(n), y_1(n), y_2(n)) = f(\bar{x}(n), y_1(n))$ are satisfied, $(x_1, y_1)$ and $(x_2, y_2)$ exhibit the same chaotic attractors [as in Fig. 2(a)] guaranteed by Lemma 1. It implies chaos synchronization. For $y(0) \neq y_2(0)$ we have Lemma 2.

Lemma 2: If $y(0) \neq y_2(0)$ and $|\bar{x}(0)| < x_a$, the ICC system has two symmetric periodic attractors if

$$F_a(x_1, 1, -1) > x_a, \quad F_a(x_b, 1, -1) < x_b.$$  

Proof: Since $F$ is continuous and is unimodal on $I_a$, $\equiv [F(x_a) - \epsilon, F(x_a) - \epsilon + \epsilon(n)$ is a small positive number], Condition (8) guarantees that the iteration $x_{n+1} = F(x_n), F : I_a \rightarrow I_a$ has an attractor in $I_a$.
That is, the system has two symmetric periodic attractors, one of which is shown in Fig. 2(c).

In this lemma, we can see an essential function of the ICC that makes stable dynamics by averaging two expanding maps with opposite slopes \(|d/df| \{ \varepsilon, 1 \} > 1\), \((d/df) \{ \varepsilon, -1 \} < -1\), and \(1/2|d/df| \{ \varepsilon, 1 \} + |d/df| \{ \varepsilon, -1 \}| < 1\) for \(x_\ast < |\varepsilon| < x_\ast\). Then Lemma 1 and Lemma 2 guarantee the coexisting phenomenon of chaos synchronization, a periodic attractor, and their symmetric ones. The parameters’ conditions (8), (10) are satisfied in the shaded region in Fig. 6.

III. CONCLUSION

Using the ICC, we have considered a simple coupling system of two nonautonomous chaotic circuits. The ICC changes the two chaotic attractors into a coexisting state of chaos synchronization, a periodic attractor and their symmetric ones. The coexisting phenomenon is guaranteed theoretically and is demonstrated in the laboratory. Now we are extending the ICC system to a coupled system of a large number of chaotic circuits and are analyzing their various synchronous phenomena. It may be developed into a novel artificial neural network.

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Recurrent Least Squares Support Vector Machines

J. A. K. Suykens and J. Vandewalle

Abstract—The method of support vector machines (SVM’s) has been developed for solving classification and function approximation problems. In this paper we introduce SVM’s within the context of recurrent neural networks. Instead of Vapnik’s epsilon insensitive loss function, we consider a least squares version related to a cost function with equality constraints for a recurrent network. Essential features of SVM’s remain, such as Mercer’s condition and the fact that the output weights are a Lagrange multiplier weighted sum of the data points. The solution to recurrent least squares (LS-SVM’s) is characterized by a set of nonlinear equations. Due to its high computational complexity, we focus on a limited case of assigning the squared error an infinitely large penalty factor with early stopping as a form of regularization. The effectiveness of the approach is demonstrated on trajectory learning of the double scroll attractor in Chua’s circuit.

Index Terms—Double scroll, radial basis functions, recurrent neural networks, support vector machines.